Mathematics in the New Zealand Curriculum – A Concept Map of the Curriculum Document

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This report analyses the *Mathematics in the New Zealand Curriculum* document and maps the strands and achievement objectives of Levels 2 to 4 into eight major categories (i.e., *understanding number*, *computing and estimating*, *time*, *metric measurement*, *shape and space*, *transformation and symmetry*, *probability*, and *understanding statistics*). Each category has one to three subcategories, giving a total of 13 subdivisions of mathematics. A full discussion and rationale for this repackaging of the curriculum is provided. A second analysis of the curriculum by cognitive process is outlined by curriculum levels. This identifies the achievement objectives at each level by the type of activity required to elicit evidence of attainment.

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Introduction

The *Mathematics in the New Zealand Curriculum* (*MiNZC*) document (Ministry of Education, 1993a) was the first of the curriculum statements released under the New Zealand Curriculum Framework (Ministry of Education, 1993b). It differed from previous syllabus statements on mathematics in format, content, emphasis, and recommended approaches. It has been used by teachers for the last eight years. The New Zealand curriculum statements are undergoing a stocktake in 2001. In this process, the experiences of classroom teachers and the findings of research in the past eight years will be considered alongside the curriculum statement. This may lead to changes in the recommended approaches or the emphases given to content areas in the curriculum document.

Purpose of the Report

The analysis presented here maps the concepts covered by the mathematics curriculum at Levels 2, 3, and 4. It covers the six strands of the curriculum and includes all of the achievement objectives at these three levels. The map is intended to inform the development of assessment tools for the asTTle (Assessment Tools for Teaching and Learning) project, showing how the achievement objectives are linked and how they reflect the development of mathematical concepts. Two maps are presented. One divides the curriculum into key themes, which simplify the sub-strands identified in the document. The second focuses on processes, and presents the objectives in clusters based on the mathematical processes they promote and the skills children develop across the strands.

The MiNZC Document and its Development

Mathematics in the New Zealand Curriculum replaced several separate curriculum documents in mathematics. In the primary area, two documents had existed previously – one for Years 0–6 (Department of Education, 1985) and one for Years 7–8 (Department of Education, 1987). It was the first document in the revision of the New Zealand curriculum to present a "seamless" statement of objectives from school entry to the end of secondary schooling. For the first time, teachers could easily trace the origins and consequences of mathematical understandings through the levels presented in the curriculum.

The Education Review Office (1994), in considering the implementation of the newly gazetted *MiNZC* document in 272 schools, listed key influences on the document's development. These were current syllabi, good classroom practice, new technology, the results of the International Association for Educational Achievement (IEA) mathematics study (1979–81) and two influential international documents – the National Council of Teachers of Mathematics (NCTM) Standards (1989) and *Mathematics Counts*, known as the Cockcroft Report (Committee of Inquiry into the Teaching of Mathematics in Schools, 1982). These two documents, the first from the United States and the second from Great Britain, influenced curriculum development in several countries. They began to articulate a constructivist approach to developing mathematical concepts, based on context-bound problem solving and a focus on children's mathematical thinking. Alongside this were measures to order and level mathematical concepts into a prescription that could be followed by classroom teachers. The strand of *mathematical processes* was introduced to promote the teaching and assessment of problem-solving processes and communication of mathematical ideas. This reflected the NCTM Standards emphasis on a problem-solving base for the curriculum.

Garden (1997) considered the results on curriculum analysis undertaken for the *Third International Mathematics and Science Study* (1995). It revealed that for "...the new curricula in maths and science, New Zealand teachers were expected to cover more topics each year than were teachers in most other countries" (p. 187). In critiquing the document, Howson (1994) noted that problems "...do not arise from the choice of educational aims, but rather from over-ambition" (p. i). Not only is the *MiNZC* document broad and full conceptually, but the recommended teaching approaches are challenging: "...implementation of them in the manner which is intended demands considerable expertise in subject matter, pedagogy and assessment" (Howson, 1994, p. 187). Howson (1994) considered that the document contained too many concepts for children to tackle, and that the constructivist approach was difficult to use well.

Since the introduction of *MiNZC*, the Ministry of Education has provided inservice support, through the Teachers or Schools Support Services Advisory and Colleges of Education (ERO, 1994). The Mathematics and Science Taskforce recommended that the Ministry provide additional support and materials for teachers. Subsequently, the Ministry has published guides to problem solving (Holton & Thomas, 1999), to the development bands for better students (Ministry of Education, 1996), and to developing mathematics programmes (Ministry of Education, 1997). They have provided booklets for teachers (*Figure it Out* and *Connected*), and there are Internet resources available through Te Kete Ipurangi (www.tki.org.nz) and 'NZ Maths' (www.nzmaths.co.nz) sites.

The work of several review groups has increased recognition of the importance of the *number* strand, particularly at Levels 1 to 3. *Update 45* (Ministry of Education, February 2001) outlines the development of frameworks for understanding children's number concepts.

This work is supported by professional development for teachers, and aims to increase the emphasis given to developing number sense and computation by promoting mental strategies and understanding their emergence.

The *MiNZC* statement has directed teachers to a new way of teaching mathematics. Over the eight years since its introduction, it has become a familiar document to classroom teachers. Resources and professional development have been provided to support its implementation in schools. Its effectiveness in practice is being currently considered in the curriculum stocktake process.

The Structure of MiNZC

The *National Education Guidelines* (Ministry of Education, 2000), The *New Zealand Curriculum Framework* (Ministry of Education, 1993b), and the *MiNZC* document all state clearly the importance of mathematical literacy for daily life and in the world of work. Mathematics is considered essential to functioning well in society, as it contributes to many daily activities (Ministry of Education, 1993b, p. 11). Mathematics taught in classrooms should reflect this by using context-based, real-life problems, according to the curriculum. Sense-making and effective problem solving are key aims.

Mathematics in the New Zealand Curriculum divides mathematics into five content strands and one processes strand. The content strands are *number*, *geometry*, *measurement*, *algebra and statistics*, and *probability*. The sixth strand is *mathematical processes*. This strand includes the sub-strands of *problem solving*, *communicating mathematical ideas*, and *logic and reasoning*, and is where thinking and working "like a mathematician" is taught. This strand is taught as an integral part of the content strands.

Each strand has eight levels, except Number which has six. Each level is designed to cover two years of schooling, although the document shows that children at a given Year may be working at a number of levels (*MiNZC,* Ministry of Education, 1993a, p. 17). The strands are each divided into sub-strands, and objectives are established for each of these sub-strands. Examples of learning activities are given on the pages facing the objectives, and there are also suggestions for assessment and extending children at each level. The objectives all begin with "within a range of meaningful contexts, children should be able to...", referring to the importance of sense-making and relevance in designing mathematics tasks.

The Concept Maps

Two concept maps are presented in this report. A concept map attempts to show graphically the development of mathematical understanding as it is outlined by the curriculum document. These maps do not go beyond the curriculum into the concepts implied by the objectives. They order and link the descriptions given in the curriculum document. At each level, the *MiNZC* document provides objectives that can be clearly linked to those at the next level. While they vary widely in specificity, the objectives follow one another logically. The achievement objectives are widely used by teachers as the basis for daily as well as unit planning.

The first map condenses these objectives into an eight-part framework. This map is based on the curriculum sub-strands, rather than on the six principal strands. The sub-strands have been condensed into key themes, and in some cases objectives have been moved from the strand in which they are found in the curriculum. These changes are described in detail below. The objectives are presented by level, showing how they develop from the previous level. In this map, mathematical process objectives are presumed to be taught across all eight parts of the framework, and are not listed separately.

The second map reorganises the curriculum objectives according to the types of mathematical activity they imply. Each of the six areas is listed with the mathematical processes that pertain to each area preceding the content objectives. This preserves the pre-eminence of process objectives, reflecting the emphasis in the curriculum.

Writing a mathematics curriculum requires decisions about what will be included, where it will sit, and how it will be described. There are commonalities between curricula internationally, but there are many differences too. Any decisions about where content objectives "belong" must be somewhat arbitrary. Mathematics is a tool for thinking, an integrated whole. The decisions about organising the maps presented here have been made with reference to classroom practice and mathematics education literature, but are still to some extent arbitrary.

The Content Map

The first map collapses the curriculum objectives into eight areas – *understanding number*, *computing and estimating*, *time*, *metric measurement*, *shape and space*, *transformation and symmetry*, *probability*, and *understanding statistics*, each reflecting key themes. While some of these areas correspond to the sub-strands in the curriculum document, others have been broadened to include objectives from several strands that contribute to the development of similar concepts. Each of the areas is discussed below. The Content Map itself can be found in Appendix One.

Understanding Number

Developing a number sense (McChesney & Biddulph, 1994), or understanding of how numbers work, is increasingly recognised as a key task for learners in New Zealand classrooms. *Understanding number* is based on the sub-strand *exploring number* in the curriculum document, and also includes objectives related to pattern from the *algebra* sub-strand. Two key themes of *understanding number* are *whole numbers* and *fractions and decimals*. These objectives are grouped in this way to help teachers trace the development of place value understandings (*whole numbers*) and proportional thinking (*fractions and decimals*). These areas are also treated as separate areas within the *Learning Framework for Number* (Ministry of Education, 2001).

Understanding number and *computing and estimating* are broadened by the inclusion of the *algebra* objectives at each level. English and Halford (1995) suggest that "understanding algebra... depends on constructing appropriate mental models of the essential concepts" (p. 70). These concepts are "primarily concerned with the relations between variables" (English & Halford, 1995, p. 70). Algebra is more than "generalised arithmetic" for this reason, and it includes conventions that make arithmetic-based thinking a disadvantage (e.g., concatenation). Nevertheless, English and Halford (1995) assert that "it does not follow that arithmetic cannot be a useful analog for algebra" (p. 72). They argue that analogues can be a powerful means of introducing concepts and acquiring an understanding of relations, which is central to progress in algebra. This aspect "has often been neglected, because we tend to test for mastery of mathematical procedures and do not test whether children understand mathematical relations" (English & Halford, 1995, p. 72).

Placing the *algebra* objectives alongside the *number* objectives, classified under separate key themes, emphasises the idea that arithmetic can be used as an analogy for algebra in these early stages. It also highlights the points at which children may become locked into one way of thinking, to the detriment of their understanding of algebra. Algebra has a crucial part to play in understanding what numbers are, what they can do, and how they do it. As children's understanding of the number system increases, both arithmetical and algebraic thinking need to be considered.

Computing and Estimating

This area of the map is derived from the *exploring computation and estimation* sub-strand of the *number* strand in *MiNZC*. Two key themes emerge in this area – *operations* and *problem solving*. This distinction is drawn because the objectives cluster around two focuses: (a) specific skill in the area of operations and (b) writing and solving problems involving operations.

Two other groups of objectives are brought into this area. Objectives relating to money from the *measurement* strand are listed here. Problems with money necessarily involve operations on numbers. Money is often used as a context for problem solving as it provides a familiar, real-life context for some children.

Objectives involving symbols and equations from the *algebra* strand are also listed here, again emphasising the interconnectedness between *number* and *algebra* at this level (English & Halford, 1995) and providing information about the symbols children might be using as they solve problems, reflecting the second key theme of *problem solving*.

Nickson (2000) noted that the "alternation of arithmetic-to-algebra and algebra-to-arithmetic pathways appears to be particularly important throughout this [early] phase of learning algebra. This helps to remind children of the connections between the two, and the changes that are involved from one to another" (p. 145).

Algebra objectives relating to the graphing of relations are also included here. Considering relations alongside arithmetic operations broadens children's often narrow conception of operations as "two numbers equalling something".

Time

Measurement objectives relating to *time* are clustered together. They form a sub-strand of the *measurement* strand, and clearly stand apart from other areas of content. These objectives deal with telling the time, in units ranging from a second to a year. Telling the time is a distinct skill that can be separated from the other *measurement* objectives. Also, the measurement of time is based on groupings that differ from the base-10 groupings used in other measurement tasks.

Metric Measurement

At Level 1, children are asked to order and compare, and to use non-standard units. The emphasis in Levels 2 to 4 is on developing the ability to use metric measurement. The key theme at these levels is that measurement should be taught through practical tasks requiring the use of measuring equipment. These objectives come from the *estimating and measuring* sub-strand of the *measurement* strand in the *MiNZC* document. Both linear measurement and measurement in three dimensions are covered as the levels progress.

Temperature is also considered here. The Celsius scale is not a metric measurement, but remains with this cluster of objectives as it requires the mastery of reading a scale with standardised units.

Shape and Space

This area has two key themes – *shape* and *position*. Most of these objectives appear in the *exploring shape and space* sub-strand of the *geometry* strand in *MiNZC*, and clearly form a cluster of concepts. At Level 4, the *measurement* objective relating to calculating area, perimeter, and volume has been included under the *shape* theme. Measurement of area, perimeter, and volume provides a mathematical way of describing a shape and its properties. Area and perimeter are often a "sticking point" for learners. Understanding these concepts in a spatial way, and not simply as counting tasks, may produce benefits for children (Nickson, 2000).

Transformation and Symmetry

This area includes a clear cluster of objectives from the *MiNZC* document. They form the *exploring symmetry and transformation* sub-strand of the *geometry* strand in the document. These objectives deal with rotation, reflection, translation, enlargement, and reduction, and with reflective and rotational symmetry.

Probability

Probability objectives are a sub-strand of the *statistics and probability* strand of the curriculum. They deal with understandings about choice and chance and how data of this sort can be generated and used predictively. They focus on the use of everyday language and experience at Level 2, and progress to more formal understandings and the use of appropriate equipment, such as spinners and dice, at Level 4. The *probability* objectives require children to think in a characteristic way about the world and the phenomena they observe; thus, in this analysis these objectives form a key theme that is distinct from the other *statistics* objectives.

Understanding Statistics

The *understanding statistics* area combines two sub-strands of the document under two key themes – *investigation* and *interpretation*. The *investigation* objective cluster details how children should be designing and carrying out statistical investigation. The *interpretation* theme covers objectives about the discussion and interpretation of their own and others' displayed data.

The objectives are grouped in the same way in the curriculum document. An additional objective, taken from the *measurement* strand at Level 4, has been clustered with them. The objective, to "design and use a simple scale to measure qualitative data", is included in the *investigation* cluster as it is a technique that needs to be used to complete activities of the types suggested by the Level 4 objectives.

Summary

The Content Map moves some *algebra* and *measurement* objectives from their places in the curriculum document. The map is otherwise very similar in structure to the curriculum document. This reflects the clarity of the organisation of the curriculum document, and the nature of the objectives. While there is obviously interdependence in the actual development of concepts (one needs to know how to read two-digit numbers to use a centimetre ruler, for example), the objectives are presented in conceptual clusters in the curriculum document. The limitations of this map are that the *mathematical processes* objectives are not treated specifically, but rather presumed to occur across the curriculum objectives; and that the broader objectives are not broken down into more manageable parts, which would make them more useful for instruction.

The Processes Map

The second map, the Processes Map, collapses the curriculum into six areas, based on the mathematical activity the learner is involved in when carrying out the tasks involved in working towards the objectives. This analysis puts the mathematical processes first, as a filter for the content objectives. The process objectives are given in the grey boxes in the map. In this map, objectives from all the content strands are mixed together, highlighting the commonalities between strands in terms of mathematical activity. Some objectives appear twice, as they include two activities – for example, *write and solve*, which uses two separate skills.

The categories were derived from an analysis of the verbs used in the objectives, which all begin "Within a range of meaningful contexts children should be able to...' – couching the objective in active terms. Activities that typically arise from these objectives were also considered, using the suggested learning experiences and commonly used resources for teaching mathematics. These further illustrate what the children are centrally being asked to do in order to achieve each of the objectives.

Each of the six categories is outlined below. The Processes Map can be found in Appendix Two.

Pose/write/create/design

This category includes objectives that ask children to be original and use their own ideas to create mathematics, encouraging them to use mathematics themselves to make something new.

Model/make/carry out

These objectives ask children to physically demonstrate or model their understandings. They require materials and hands-on activity to complete.

Tell/show/explain

This cluster highlights the curriculum's emphasis on explaining mathematical understandings. These objectives ask children to explain or demonstrate their understandings in a range of ways.

Know

Some specific knowledge items and end points are defined by the curriculum. These are clustered here as things that children should know at given levels.

Read/follow

Children are required to be able to interpret and read as well as to produce mathematics. These objectives ask children to receive mathematical information by reading or following directions.

Solve

This category highlights the curriculum's emphasis on problem solving. Children are asked to solve problems across the content strands, and this cluster shows how these objectives are all related to problem solving processes.

Discussion

This report presents two concept maps of the *MiNZC* objectives at Levels 2, 3, and 4. The maps each highlight different aspects of the document. Three key areas for discussion arise from this: teaching issues, organisation of the objectives, and the development of mathematical concepts.

The document directs *how* mathematics should be taught, as well as providing the mathematics content to be taught. Teaching issues arise from this. Objectives in the MiNZC document are written and organised in certain ways, which affects both how the concepts are taught and how they are assessed. Finally, MiNZC attempts to organise children's cognitive development into a framework for teaching. This has important implications for teachers and learners. These issues are discussed below.

Issues for Teaching from MiNZC

General themes of the MiNZC document. Several general themes underlie all the achievement objectives in the curriculum document. These are featured in the introduction to the curriculum. Mathematics is seen as vital to effective participation in society and work. It should be relevant to children and therefore must be presented in realistic contexts. Mathematical tasks should encourage children to act like mathematicians – using conjecture and strategy to solve problems. Activities should therefore encourage thinking and the use of problem-solving processes. The best activities will cover several objectives, and will be mathematically rich as well as mathematically authentic.

Establishing mathematical concepts will involve the use of appropriate equipment and active manipulation by children. Confidence in and enjoyment of mathematics will be promoted.

New material at earlier levels. Mathematics in the New Zealand Curriculum (*MiNZC*) (Ministry of Education, 1993a) introduced some concepts at earlier stages of schooling than previous curricula had done. Algebra and statistics in particular were now required from the beginning of schooling. The development of understanding in these strands, while it tends to appear clearly defined, may not reflect the ways in which children learn in these areas. An example of this is the *probability* objectives, which appear to be in a clear, logical sequence. However, Nickson (2000) found that the possible contributions of both developmental stages and children's intuition to understanding probability needed to be considered in determining the sequence of probability objectives. Underlying cognitive structures and strategies in primary-age children are less well understood in content strands such as *statistics* than in the area of *numeracy*.

Mathematical processes strand. The sixth strand of *MiNZC* covers *mathematical processes*, which has a different structure from the content strands. *Mathematical processes* includes three sub-strands (*problem solving*, *communicating mathematical ideas*, and *logic and reasoning*), and objectives are listed for each of these. Some objectives are introduced at later levels, while others are the same for all levels.

The strand attempts to define the skills of working mathematically that should be taught across the content strands. The objectives necessarily represent a selection of the skills required to "do mathematics", and they have been assigned to levels by the curriculum writers. It is possible to see *communicating mathematical ideas* and *logic and reasoning* as contributors to the *problem solving* sub-strand, making *problem solving* the overall thrust of the *mathematical processes* strand. This suggests that interpretations of this strand are required in order to make it useful for the classroom.

When the curriculum was first introduced, there was a tendency to approach this strand by "doing problem solving on a Friday" or giving problem-solving activities for homework (ERO, 1994). *Mathematical processes* should be embedded in the approach taken to teaching the content strands. The Content Map assumes that this is so, and does not specifically list the *mathematical process* objectives. The Processes Map, however, shows the processes as a "filter" for activity across the content strands.

The way in which the content objectives of the curriculum are worded implies a particular approach to teaching. The Processes Map highlights this by showing which type of activity the achievement objectives imply, and linking this to the *mathematical processes* objectives. The Processes Map clearly illustrates two key points. Firstly, problem-solving activities will better cover *mathematical processes* objectives than will rote learning activities. This is demonstrated by the number of processes covered by each of the six categories. Secondly, the curriculum's emphasis at each level is shown by the balance of objectives in each of the activity categories. At Level 2, for example, the emphasis is on problem solving and explaining mathematical ideas. At Level 4, children are required to have more knowledge or formal understanding, as well as to solve problems.

The Processes Map could not form the basis of a reporting system, as the activity categories do not give enough information about achievement objectives – for example, progress in explaining ideas would cover many different content areas, and valuable information about each would be lost. However, it serves as an adjunct to the Content Map, where processes are implied and the overall character of the document is not described.

Planning and reporting. Since the introduction of the curriculum document in 1993, several resources have been produced by advisers and publishers to help teachers interpret the curriculum. Planning is largely based on a strand-by-strand teaching programme, involving maintenance of prior learning and the introduction of new concepts, often in ability groups. Reporting to parents and colleagues is often organised in a strand-by-strand manner, based on achievement objectives from the curriculum document.

The objectives drawn from the curriculum are often used as checklists, despite their unsuitability for this in some cases. Planning and teaching follow the objectives closely, using strands and sub-strands as reference points. Any feedback to teachers on their children's performance must therefore be clearly related to the objectives they have selected for teaching. The Content Map places the *algebra* strand with the *number* objectives that feed into it at this level, as discussed earlier. For the purposes of reporting, however, algebra would need to be offered as an option to teachers who may be teaching an algebra unit and require an assessment for these objectives alone. These objectives are identified within the map using the letter code, but would need to appear in full in an assessment resource for teachers.

Objectives in MiNZC

Breadth of objectives. The Content Map gives a linear analysis of the development of concepts expressed in the achievement objectives at Levels 2, 3, and 4. It links each objective to the prior objectives from which it "grows", and to the understandings it is heading towards. These achievement objectives, however, raise several issues which make this linear connection problematic. Firstly, some of the objectives are very specific, while others are very broad. The objectives relating to probability appear specific and to follow a clear progression (Table 1).

Table 1
Probability objectives

Compare events and order by likelihood (S L2-4)
Use a systematic approach to count possible outcomes (S L3-5)
Predict likelihood based on observations (S L3-6)
Use tree diagrams to find all possible outcomes (S L4-9)
Estimate frequencies and mark on a scale (S L4-8)

For these objectives, it is easy to observe children's performance and categorise it by level. In response to a probability problem, a Level 3 child might draw a series of pictures to explore all of the possible outcomes, while a Level 4 child might use a tree diagram to find all the possible outcomes. The objectives are clearly heading from the use of informal discussion related to the child's experience towards formal techniques for exploring probability.

In contrast, some objectives are very broad and ill-defined. The most significant of these, because of their centrality to the development of children's numeracy, are the *write and solve* objectives within the *number* strand.

Table 2
Story problem objectives

Write and solve story problems – with whole numbers
– with 1 operation (N L2-10)
– using any combination (N L2-11)
Write and solve story problems– with fractions – 1/2s,1/3s,1/4s,1/5s (N L2-5)
Write and solve story problems – with whole numbers and decimals
– any combination of operations (N L3-6)
Write and solve story problems – with decimal multiplication and division (N L4-8)

Although the parameters of the children's problem solving are defined (the operations and types of numbers are given), there is a huge amount of mathematics implied by these objectives. If they are taken as written, they mean that children at Level 2 should be able to write and solve problems involving multiplication and division in combination, using *any* whole number – no account is taken of the complexity of the calculation (e.g., number of digits in the numbers). In practice, each teacher or school has defined what will be seen as competence within these objectives, potentially resulting in considerable variation between schools.

The Level 3 and 4 objectives given in Table 2 illustrate a further difficulty. The Level 3 objective implies that children should be able to do decimal multiplication and division, yet the Level 4 objective specifies this as a Level 4 skill. Again, finding out where each of these competencies sit in practice has been left to teachers to interpret.

Two-part objectives. Many of the achievement objectives include two parts – *write and solve*, *model and describe*, for example. Each element of these pairs is a separate activity, requiring distinct skills. While these paired objectives mirror one another, solving a problem and writing a problem, for example, call on different sets of skills.

The Content Map leaves the objectives intact, further broadening their scope. The Processes Map separates these objectives into implied activity categories, thus highlighting the different competencies implied by each one. Again, using the Processes Map as an adjunct to the Content Map will give a fuller picture of the implications of the curriculum document.

Interconnectedness of concepts. Many of the objectives that have been organised into clusters in this analysis can be seen to "feed" one another. Understanding about the nature of shapes and their properties is necessary in order to understand how to calculate perimeter and area, for example. Similarly, understanding the base-10 place value system leads to a meaningful understanding of the metric system for measurement. And in order to sensibly interpret graphs, a child needs to understand the quantities represented and know how these can be compared.

While valid divisions can be defined within the mathematics curriculum, children's mathematics understandings are built across these categories. Ideally, concepts would be added to and applied across the curriculum strands. As strands are often used as a basis for planning and teaching, making these connections explicit becomes a key task for teachers.

MiNZC and the Development of Mathematical Concepts

A period of major change. Levels 2, 3, and 4 of the curriculum cover a time of substantial change and development in children's mathematics. These levels cover the emergence of place value understandings and their extension into decimals. They include the development of part-whole concepts and proportional thinking. In addition, children move from considering simple shapes in their environment to considering more formal geometry (e.g., knowing the symmetries of regular polygons, using diagrams to represent objects) and from the concrete to the abstract in measurement.

Essentially, across the strands, children move from informal description of their own experience to mathematical description of abstractions. The achievement objectives from Levels 2 to 4 represent a fundamental shift, which children need to make successfully if they are going to go on to understand higher-level mathematics.

The curriculum and developments in the Learning Framework for Number. Update 45 (Ministry of Education, 2001b) and the *Education Gazette* (5 March 2001) outline the Ministry of Education's initiative in numeracy. Based on the *Count Me In Too* programme (New South Wales Department of Education and Training, 1999), a *Learning Framework for Number* has been developed. This framework is being explored with teachers through a large inservice project.

The work is currently being extended from the first three years of school to Years 4–6 in a project called the *Advanced Number Project* (Ministry of Education, 2001a). The Advanced Number Project (ANP) proposes a *maunga*, or mountain, of increasingly sophisticated strategies that children use to solve problems. The ways in which children approach problems can be used to characterise their understandings and to establish what they should be taught next.

This number framework currently sits alongside the official curriculum document. The Ministry of Education intends to align its resources with the number framework (Velde, 2001), and the curriculum document itself is currently undergoing a stocktake process. Integrating the *Learning Framework for Number* and the curriculum document could be done in many ways. Two suggestions are given here, in the light of the foregoing analysis.

Firstly, as one of the intentions of the number framework is to identify and teach children to use a range of strategies to solve problems, this range of strategies could be considered against every achievement objective. One of the findings of the *Advanced Number Project* (Ministry of Education, 2001a) is that individual children tend to use a characteristic approach to solving problems across the number frameworks categories (addition/ subtraction, base 10, multiplication, fractions). Performance on problems relating to the *number* achievement objectives should yield a personal pattern for any particular child.

Because a central assumption of the ANP is that children develop and select their own strategies, strategies could be discovered through this process, rather than taught to match each objective. The teacher's role would then be to move children on to more powerful strategies which enable them to process higher-level mathematics.

A second way to integrate the *Learning Framework for Number* and the achievement objectives is to consider the mathematical structures behind the achievement objectives and define which strategies children would need to use in order to understand each objective in a way that will allow them to progress.

While strategies such as counting on can be very versatile and will take children through many mathematical situations, there comes a time when the cognitive load of counting on makes it impractical (e.g., handling large numbers, multiplication), or when counting on does not give the learner sufficient power to solve problems. Proportional reasoning becomes particularly important in understanding fractions and decimals and some strategies could be seen as prerequisite to operating effectively at Level 4 of *MiNZC*.

This approach would need considerable research and discussion, as the notion of a range of strategies at each stage is pivotal to the *Learning Framework for Number*. In terms of higher mathematics, however, there may be some ways of thinking that are more powerful than others, and some of these may be necessary in order to grasp later concepts. Linking certain strategies with specific achievement objectives at each level may bring these two analyses together.

Strategies, knowledge, skills, and understandings. Objectives within Levels 2 to 4 of the mathematics curriculum specify both things that students should *know* and things that students should *do*. The terms *knowledge* and *skills* are often used respectively to describe these, but this terminology is problematic. The curriculum links the two terms: "...while such skills [algorithms] are important, a consequence of a narrow assessment regime which isolates discrete skills or knowledge is that students tend to learn in that way" (Ministry of Education, 1993a, p. 15).

This leads to difficulties. If a child knows the algorithm for two-digit addition, is this a skill they have, knowledge they possess, or a strategy for solving appropriate problems? Do skills become knowledge when they are automatic? Is a strategy a skill, or is it knowledge? The *Advanced Number Project* identifies strategies and knowledge as two components of numeracy, and promotes them alongside each other, noting that certain knowledge is necessary in order to develop strategies, and vice versa (Ministry of Education, 2001a).

This debate becomes important when we consider the curriculum in terms of a concept map. Are we mapping the acquisition of knowledge or the emergence of skills? We might consider the *mathematical processes* strand to cover the skills necessary for mathematics, and the content strands to cover the knowledge, but this is an over-simplification. Holton (1999) recommends the explicit teaching of problem-solving strategies: this leads to the question of whether problem-solving strategies when mastered become skills or knowledge.

"Knowing about" and "doing" mathematics are bound together in a reciprocal relationship. Recognising this reciprocity in planning, teaching, and assessment is important to ensure balanced development of concepts.

Summary

This report presents two maps of the *Mathematics in the New Zealand Curriculum* (*MiNZC*) document (Ministry of Education, 1993a). The first map traces the development of achievement objectives from Level 2 to Level 4, indicating within eight broad thematic areas how the objectives are linked. The second map serves as an adjunct to the first, giving an overview of each level. Objectives are organised in terms of process skills and activity types, while the content strands are not separated out. This map shows how the process objectives can be viewed across the five content strands, and how the prescription at each level is balanced in terms of these processes.

The objectives drawn from the curriculum correspond to a period of rapid cognitive growth and change in understandings about mathematics. This has been explored by the new numeracy initiatives, which have developed a strategy and knowledge framework for the primary school years. Objectives, particularly in the *number* strand, include many concepts and skills that are not itemised. This has many implications for assessment.

The *Mathematics in the New Zealand Curriculum* document has been used as a basis for planning and assessment by teachers, who often use the broad objectives set out within the document as checklist items. Schools have worked independently on developing criteria for assessing performance at the various levels, and various publications have also sought to help with this. The apparent clarity of the document belies the complex understandings implied by the achievement objectives.

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Appendix One: Content Map

Appendix Table 1

Understanding Number

Key Themes	Level 2	Level 3	Level 4
Whole Numbers	Read 3-digit numbers (N L2-1)	Read any whole number (N L3-1)	
	Explain meaning of 2- or 3-digit numbers (N L2-2)	Explain any whole number (N L3-1)	Explain meaning of negative numbers (N L4-1)
	Order any three 2-digit numbers (N L2-3)	Order any whole number (N L3-3)	
			Explain meaning of and evaluate powers of whole numbers (N L4-2)
Pattern in Number	Continue a sequential pattern and describe a rule for this (A L2-1)	Describe in words a rule for number and spatial sequential patterns (A L3-1)	Find a rule to describe any member of a number sequence (words) (A L4-1)
		Make up and use a rule to create a sequential pattern (A L3-2)	Use a rule to make predictions (A L4-2)
Fractions and Decimals	Use 1/2s,1/4s, 1/3s and 1/5s in story problems (N L2-5)	Find fractions of whole numbers or decimal amounts (N L3-7)	Express quantities as fractions or percentages of a whole (N L4-6)
	Represent sums of money with 2 or more combinations of notes and coins (M L2-3)		Find equivalent fractions (N L4-3)
		Explain meaning of digits to 3 decimal places (N L3-2)	Convert fractions to decimals and decimals to fractions (N L4-4)
			Convert decimals to percentages and percentages to decimals (N L4-5)
		Order numbers up to 3 decimal places (N L3- 2)	

Appendix Table 2

Computing and Estimating

Key Themes	Level 2	Level 3	Level 4
Operations	Mentally perform addition and subtraction (N L2-8)		
	Recall basic addition/subtraction facts (N L2-7)		
	Use multiplication facts (N L2-9)	Recall multiplication facts (N L3-5)	
	Add, subtract, multiply, divide whole numbers (N L2-10)	Add, subtract, multiply, divide decimal and whole numbers (N L3-6)	Explain satisfactory algorithms for addition, subtraction, multiplication (N L4-10)
	Give change for sums of money (M L2-2)		
	Use =, >, < (A L2-3)	Solve problems of the type x + 15 = 39 (A L3-5)	Solve simple linear equations such as 2x + 4 = 16 (A L4-5)
			Demonstrate knowledge of order of operations (N L4-11)
	Use graphs to illustrate relationships (A L2-2)	Use graphs to represent number or informal relations (A L3-4)	Sketch and interpret graphs on whole number grids showing everyday situations (A L4-3)
Problem Solving	Write and solve comparison problems (N L2-4)		
	Make sensible estimates and check reasonableness of answers (N L2-6)	Make sensible estimates and check reasonableness of answers (N L3-4)	Make sensible estimates and check reasonableness of answers (N L4-7)
	Write and solve story problems – with whole numbers – with 1 operation (N L2-10) – using any combination (N L2-11)	Write and solve story problems – with whole numbers and decimals – any combination of operations (N L3-6)	Write and solve story problems – with decimal multiplication and division (N L4-8)
	Write and solve story problems – with fractions – 1/2s,1/3s,1/4s,1/5s (N L2- 5)		
		Solve practical problems finding fractions of whole numbers and decimals (N L3-7)	Find a given fraction or percentage of a quantity (N L4-9)
		State the rule for a similar set of practical problems (A L3-3)	Find and justify a word formula which represents a practical situation (A L4-4)

Appendix Table 3
Time

Key Theme	Level 2	Level 3	Level 4
Measuring Time	Read time (M L2-4) – digital and analogue clocks Know units of time (M L2-4) – minute, hour, day, week, month, year	Convert digital to analogue and vice versa (M L3-4) Read and interpret everyday statements about time (M L3-3)	Perform calculations with time, including 24-hour clock (M L4-5) Read and construct a variety of scales, timetables and charts (M L4-3)

Appendix Table 4
Metric Measurement

Key Theme	Level 2	Level 3	Level 4
Practical Measuring	Carry out practical tasks, using appropriate metric measures for: – length – mass – capacity (M L2-1)	Make estimates with units of: – length – mass – volume – area – temperature (M L3-1) Perform measuring tasks with a range of units and scales (M L3-2)	Read a scale to the nearest gradation (M L4-1)

Appendix Table 5

Shape and Space

Key Themes	Level 2	Level 3	Level 4
Shape	Make, name, and describe shapes and objects using – own language – language of geometry (G L2-1)	Describe 2-D and 3-D using the language of geometry (G L3-1)	Construct triangles and circles using instruments (G L4-1)
			Calculate: – perimeters of circles, rectangles, triangles – areas of rectangles – volumes of cuboids from length measurements (M L4-2)
		Model and describe 3-D objects from diagrams/pictures (G L3-3)	Make model from top, side, front, back diagrams (G L4-3)
		Draw pictures of 3-D objects (G L3-4)	Draw diagrams of objects made from cubes (G L4-4)
		Design and make containers to specified requirements (G L3-2)	Design a net and construct a simple polyhedron to specified dimensions (G L4-2)
Position	Describe and interpret position using direction and distance (G L2-2)	Draw and interpret scale maps (G L3-5)	Specify location using bearings or grid references (G L4-5)

Appendix Table 6

Transformation and Symmetry

Key Theme	Level 2	Level 3	Level 4
Transformation and Symmetry	Make clockwise and anti-clockwise turns (G L2-4)		
	Create and talk about geometric patterns – repeating patterns – with rotational symmetry – with reflective symmetry (G L2-3)	Design and make a pattern with – translation – reflection – rotation (G L3-7)	
		Describe patterns with – reflective symmetry – rotational symmetry – translations (G L3-6)	Describe reflective or rotational symmetry of a figure or object (G L4-7)
			Apply symmetries of regular polygons (G L4-6)
		Enlarge to scale on a grid (G L3-8)	Enlarge or reduce in 2-D, note invariant properties (G L4-8)

Appendix Table 7

Probability

Key Theme	Level 2	Level 3	Level 4
Probability	Compare events and order by likelihood (S L2-4)	Use a systematic approach to count possible outcomes (S L3-5) Predict likelihood based on observations (S L3-6)	Use tree diagrams to find all possible outcomes (S L4-9) Estimate frequencies and mark on a scale (S L4-8)

Appendix Table 8
Understanding Statistics

Key Themes	Level 2	Level 3	Level 4
Investigation	Collect and display whole number data – pictograms – tally charts – bar charts (S L2-1)	Collect and display discrete numeric data – stem and leaf – dot plot – strip graphs (S L3-2)	Choose and construct data displays to communicate significant features: – frequency tables – bar charts – histograms (S L4-3) Collect and display time-series data (S L4-4) Design and use a simple scale to measure qualitative data (M L4-4)
		Plan an investigation of an assertion (S L3-1)	Plan investigation of an issue or interest (S L4-1) Collect appropriate data (S L4-2)
Interpretation	Talk about own displays (S L2-2)	Use own language to discuss – features – outliers – clusters in own and others' data displays (S L3-3)	Report distinctive features (S L4-5)
	Make statements about others' displays (S L2-3)	Make sensible statements about an assertion based on statistical evidence (S L3-4)	Evaluate others' interpretations (S L4-6) Make statements about implications and actions consistent with results (S L4-7)

Appendix Table 9

Processes Maps for Level Two

Pose / write / create / design	Model / make / carry out	Tell / show / explain
Pose questions for mathematical exploration Devise a set of instructions	Devise and use problem-solving strategies Use equipment appropriately	Explain mathematical ideas Record, in an organised way, and talk about results of mathematical exploration Interpret information and results in context
– write comparison problems (N L2-4) – write story problems 1/2, 1/4, 1/3, 1/5 – write story problems with whole numbers and +, -, x, ÷ (N L2-10) – story problems with a choice or combination of operations (N L2-11) – create geometric patterns that repeat/reflective-rotational symmetry (G L2-3)	– practical measuring tasks - length, mass, capacity (M L2-1) – make everyday shapes and objects (G L2-1) – make clockwise/anticlockwise turns (G L2-4) – collect data (S L2-1)	– meaning of 3-digit numbers (N L2-2) – order 3-digit numbers (N L2-3) – use multiplication facts (N L2-9) – describe everyday shapes and objects (G L2-1) – describe position (G L2-2) – talk about geometric patterns that repeat/reflective-rotational symmetry (G L2-3) – use graphs to illustrate relationships (A L2-2) – display data in pictograms, tally charts, and bar charts (S L2-1) – talk about their displays (S L2-2)
Know	Read / follow	Solve
Explain mathematical ideas	Interpret information and results in context Follow a set of instructions for mathematical activity	Classify objects Use words and symbols to describe and continue patterns Devise and use problem-solving strategies Use equipment appropriately
– mentally perform addition calculations (N L2-8) – basic addition and subtraction facts (N L2-7) – units of time (M L2-4) – name everyday shapes and objects (G L2-1) – use the symbols =, <, > (A L2-3)	– read 3-digit numbers (N L2-1) – make sensible estimates and check the reasonableness of answers (N L2-6) – read time (M L2-4) – interpret position (G L2-2) – make sensible statements about a data display drawn by others (S L2-3)	– solve comparison problems (N L2-4) – solve story problems 1/2, 1/4, 1/3, 1/5 (N L2-5) – solve story problems with whole numbers and +, -, x, ÷ (N L2-10) – solve story problems with a choice or combination of operations (N L2-11) – give change for sums of money (M L2-2) – represent a sum of money by two or more combinations (M L2-3) – continue a sequential pattern and describe a rule for this (A L2- 1) – compare events on a scale from least likely to most likely (S L2-4)

Appendix Table 10

Processes Maps for Level Three

Pose / write / create / design	Model / make / carry out	Tell / show / explain
Pose questions for mathematical exploration Devise a set of instructions Effectively plan mathematical exploration	Devise and use problem-solving strategies Use equipment appropriately	Explain mathematical ideas Record, in an organised way, and talk about results of mathematical exploration Interpret information and results in context
– write problems which involve whole numbers and decimals and which require a choice of the four arithmetic operations (N L3-6) – design containers to specified requirements (G L3-2) – design a pattern which involves translation, reflection, or rotation (G L3-7) – make up and use a rule to create a sequential pattern (A L3-2) – plan a statistical investigation of an assertion about a situation (S L3-1)	– perform measuring tasks, using a range of units and scales (M L3-2) – make containers to specified requirements (G L3-2) – model 3-D objects illustrated by diagrams or pictures (G L3-3) – draw pictures of simple 3-D objects (G L3-4) – draw simple scale maps (G L3-5) – make a pattern which involves translation, reflection, or rotation (G L3-7) – enlarge, on grid paper, simple shapes to a specified scale (G L3-8) – collect discrete numeric data (S L3-2)	– explain the meaning of the digits in any whole number (N L3-1) – explain the meaning of digits in decimal numbers with up to 3 decimal places (N L3-2) – order decimals with up to 3 decimal places (N L3-3) – demonstrate knowledge of the basic units of length, mass, area, volume, and temperature by making reasonable estimates (M L3-1) – show analogue time as digital time and vice versa (M L3-4) – describe the features of 2-D and 3-D objects, using the language of geometry (G L3-1) – describe 3-D objects illustrated by diagrams or pictures (G L3-3) – describe patterns in terms of reflection and rotational symmetry and translations (G L3-6) – describe in words a rule for continuing number and spatial sequential patterns (A L3-1) – use graphs to represent number or informal relations (A L3-4) – display discrete numeric data in stem-and-leaf graphs, dot plots, and strip graphs, as appropriate (S L3-2) – use their own language to talk about the distinctive features (S L3-4) – make sensible statements about an assertion on the basis of the evidence of a statistical investigation (S L3-4)

(continues)

Know	Read / follow	Solve
Explain mathematical ideas	Interpret information and results in context Follow a set of instructions for mathematical activity	Classify objects, numbers and ideas Use words and symbols to describe and continue patterns Devise and use problem-solving strategies Use equipment appropriately Effectively plan mathematical exploration
– recall the basic multiplication facts (N L3-5)	– make sensible estimates and check the reasonableness of answers (N L3-4) – read and interpret everyday statements involving time (M L3-3) – interpret simple scale maps (G L3-5) – interpret data displays, noting distinctive features (S L3-3, S L3-4)	– solve problems which involve whole numbers and decimals and which require a choice of the four arithmetic operations (N L3-6) – solve practical problems which require finding fractions of whole number and decimal amounts (N L3-7) – state the general rule for a set of similar practical problems (A L3-3) – solve problems of the type $x + 15 = 39$ – use a systematic approach to count a set of possible outcomes (S L3–5) – predict the likelihood of outcomes on the basis of a set of observations (S L3-6)

Appendix Table 11

Processes Maps for Level Four

Pose / write / create / design	Model / make / carry out	Tell / show / explain
Pose questions for mathematical exploration Devise a set of instructions Effectively plan mathematical exploration Make conjectures in a mathematical context	Devise and use problem-solving strategies Use equipment appropriately	Explain mathematical ideas Interpret information and results in context Make conjectures in a mathematical context Use words and symbols to describe and continue patterns Record information in ways that are helpful for drawing conclusions and making generalisations Report the results of mathematical explorations concisely and coherently
– write problems involving decimal multiplication and division (N L4-8) – construct a variety of scales, timetables, and charts (N L4-3) – design and use a simple scale to measure qualitative data (M L4-4) – design the net to make a polyhedron to specified dimensions (G L4-2) – plan a statistical investigation arising from the consideration of an issue or an expression of interest (S L4-1)	– carry out measuring tasks involving reading scales to their nearest gradations (M L4-1) – construct triangles and circles, using appropriate drawing instruments (G L4-1) – make a simple polyhedron (G L4-2) – make a model of a solid object from diagrams (G L4-3) – draw diagrams of solid objects made from cubes (G L4-4) – enlarge and reduce a 2-D shape (G L4-8) – sketch graphs on whole number grids which represent simple everyday situations (A L4-3) – collect appropriate data (S L4-2) – collect time-series data (S L4-4)	– explain the meaning of negative numbers (N L4-1) – explain the meaning of the powers of whole numbers (N L4-2) – explain satisfactory algorithms for addition, subtraction, and multiplication (N L4-10) – specify location, using bearings or grid references (G L4-5) – describe the reflection or rotational symmetry of a figure or object (G L4-7) – identify invariant properties of enlargements or reductions (G L4-8) – make statements about implications and possible actions consistent with the results of a statistical investigation (S L4-7) – report the distinctive features of data displays (S L4-5) – display time-series data (S L4-4) – choose and construct quality data displays to communicate significant features in measurement data (S L4-3)

(continues)

Know	Read / follow	Solve
Explain mathematical ideas	Interpret information and results in context Follow a set of instructions for mathematical activity	Classify objects, numbers, and ideas Devise and use problem-solving strategies Use equipment appropriately Effectively plan mathematical exploration Find, and use with justification, a mathematical model as a problem-solving strategy Make conjectures in a mathematical context Use words and symbols to describe and generalise patterns
– evaluate the powers of whole numbers (N L4-2) – express a fraction as a decimal and vice versa (N L4-4) – express a decimal as a percentage and vice versa (N L4-5) – express quantities as fractions or percentages of a whole (N L4-6) – find a given fraction or percentage of a quantity (N L4-9) – satisfactory algorithms for addition, subtraction, and multiplication (N L4-10) – demonstrate knowledge of the conventions for order of operations (N L4-11) – calculate perimeters of circles, rectangles, and triangles, areas of rectangles, and volumes of cuboids from measurements of length (M L4-2)	– make sensible estimates and check the reasonableness of answers (N L4-7) – read a variety of scales, timetables, and charts (N L4-3) – interpret graphs on whole number grids which represent everyday situations (A L4-3) – evaluate others' interpretations of data displays (S L4-6)	– find fractions equivalent to one given (N L4-3) – solve problems involving decimal multiplication and division (N L4-8) – perform calculations with time, including 24-hour clock times (M L4-5) – find a rule to describe any member of a number sequence and express it in words (A L4-1) – use a rule to make predictions (A L4-2) – find and justify a word formula which represents a given practical situation (A L4-4) – solve simple linear equations such as $2x + 4 = 16$ (A L4-5) – find all possible outcomes for a sequence of events, using tree diagrams (S L4-9) – estimate the relative frequencies of events and mark them on a scale (S L4-8)