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Assessment Tools for Teaching and Learning

Mathematics Curriculum Framework and Map:

Levels 2 - 6

Submitted by the Assessment Tools for Teaching and Learning team,

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This report reviews the Level 2-4 asTTle mathematics framework and identifies and discusses changes that need to be made in order to extend the framework to curriculum Levels 5 and 6 and provides a detailed breakdown of curriculum achievement objectives by curriculum level within the context of the revised asTTle mathematics framework. Changes are largely related to the place and nature of algebra, irrational numbers, and graphing in the curriculum. asTTle is funded by the Ministry of Education to Auckland Uniservices at the University of Auckland to research and develop an assessment application for Reading, Writing, Mathematics, Panui, Pāngarau, and Tuhituhi for Years 5-7 (Levels 2-4) for New Zealand schools. We acknowledge this funding, and thank the Ministry of Education for their continued assistance in the development of this project.

We especially acknowledge the leadership that Dr. Gill Thomas has brought to this review. Gill coordinated the team that analysed the curriculum in order to fit the requirements of the asTTle tool. Professor Derek Holton has worked closely with Gill over many years developing a deep understanding of the essential 'big ideas' of mathematics education for use in New Zealand schools. Andrew Tagg has worked with Gill and Derek and has provided careful checking of the wording of all the achievement objectives. Dr. Gavin Brown has worked closely with the development team, responding to their ideas, providing details on the asTTle development, and reflecting on how the curriculum map affects the asTTle development.

This report is the latest in a series of reports on the mathematics curriculum to which a wide range of curriculum experts have responded, whom I would also like to thank for readily providing feedback and analysis.

Professor John Hattie
Project Director, asTTle
Month Year

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Assessment of curriculum content requires a clear understanding of the structure and nature of the content to be assessed. Two powerful tools in guiding the development of assessment items are a content Framework and a Curriculum Map. A content Framework classifies the content of a subject into major categories or big ideas and sub-divides the big ideas into major or key themes. What is being taught in classrooms ought to relate to children learning these major ideas and related themes. Furthermore, a curriculum will specify a multiplicity of achievement objectives across a range of difficulty levels that need to be taught and mastered as the substance of the major curricular themes and ideas. A curriculum map relates the detailed achievement objectives to the major categories and key themes identified in the content Framework. The whole process of identifying the structures and relationships of curriculum content is called curriculum mapping.

This report outlines the history and structure of the asTTle Mathematics Framework and its related detailed Curriculum Map (Table 1). This document describes the major features of the revised Mathematics Framework (Table 2), provides detailed breakdowns of achievement objectives by curriculum levels (Appendix 1), and discusses the rationale for the recommended changes introduced in this curriculum map.

The asTTle Mathematics Framework and Curriculum Map as they presently stand are the result of three iterations. Ell (2001) analysed the strands and achievement objectives of levels 2 to 4 of Mathematics in the New Zealand Curriculum (1992) and produced a framework with 8 major categories and 13 key themes. An independent expert advisory panel revised this draft framework prior to the original item signature study research (Thomas, Tagg, Holton, & Brown, 2002). Discussion of the big ideas and underlying patterns resulted in relabelling and restructuring the mathematics content achievement objectives in the Ell curriculum framework into 8 categories and 20 key themes. These are in Thomas, Tagg, Holton, & Brown (2002) and are the original version of the Mathematics Framework (see Table 1) that was used for the Level 2 to 4 version of the asTTle assessment tool. The Framework provided a high-level summary of significant ideas within the curriculum document. This was complemented by a detailed curriculum map that mapped each curriculum achievement objective to a theme and category in the Framework. With the extension of the asTTle numeracy assessments to Levels 5 and 6, the Mathematics Framework was again revised (Table 2) and now has 8 major categories and 21 key themes (Table 2). It needs to be noted that the number of major categories (8) was a logistical requirement of the asTTle tool.

Basic Philosophy

The underlying philosophy behind this version of the asTTle mathematics framework and the earlier one was similar. First, the Framework and Map had to be based on Mathematics in the New Zealand Curriculum (1992). This is simply because all mathematics teaching in this country is based upon this document. Second, they had to be consistent with what are generally accepted as the big ideas of mathematics as expressed in the curriculum document as well as in similar documents from other countries and international assessment regimes. In this way the Map would have international credibility. Hence the United States NCTM Standards documents (1989 & 2000), the Australian statement (1990) and the TIMSS framework (2001), were all considered in some depth. And third, the inherent development or progression of material in the different sections of mathematics

should be incorporated into the Map. This helps the documents to be consistent with the National Exemplar Project (Ministry of Education, 2003). In this way teachers can appreciate how their students arrived at their current mathematical state and what they might master next all against a backdrop of what school students might reasonably be expected to achieve eventually.

Structure of the asTTle Mathematics Framework

The curriculum document has 6 strands: Algebra; Geometry; Mathematical Processes; Measurement; Number; and Statistics. The first dissection of the curriculum was into 4 parts based on the strands. It was felt difficult to separate the Mathematical Processes from the content areas of the curriculum. Indeed, MiNZC states that "the mathematical processes strand is deliberately intended to encourage teachers and students to make connections between the other strands wherever possible" (p. 16). Hence these four parts were derived directly from the content strands. A similar decision was made in the mathematics section of the National Exemplar Project (Ministry of Education, 2003). It was further felt that at levels 2 to 6, there was not always a clear distinction between Number and Algebra and that the latter developed naturally from the former. To highlight this, the first strand has been renamed Number to Algebra. The other three parts are the remaining strands of Geometry, Measurement, and Statistics.

The eight major categories developed naturally from here. Both Number and Geometry appear to have knowledge that can be applied, and operations that involve application of this knowledge. For instance, on the one hand explaining the meaning of digits in decimals is a matter of knowledge and so is to be found in Number Knowledge, while, expressing fractions as decimals is an application of this idea and is to be found in Number Operations. Similarly, knowing about simple angles fits into Geometric Knowledge, whereas using angles in the tessellation of shapes fits into Geometric Operations. So these provided four of the major categories found in the asTTle Mathematics Framework and Curriculum Map; Number Knowledge, Number Operations, Geometric Knowledge, and Geometric Operations.

The third major category of Number to Algebra is Algebra, while Measurement is a small but important strand at this level that stands by itself as a major category. The strand Statistics has two facets that are somewhat distinct at the school level. These are Statistics itself and Probability. By Statistics here we mean the investigating of situations. This requires the planning of investigations, and the collecting, analysing, presenting, and interpreting of data. This has a different flavour from Probability, which is much more about assigning numerical values to specific chance events. Hence the final two categories are Probability and Statistics.

Table 1.
Mathematics Framework (Levels 2-4)

Number & Algebra			Measurement	Geometry		Statistics	
Number Knowledge	Number Operations	Patterns in Number	Measurement	Geometric Knowledge	Geometric Operations	Probability	Statistics
Integers Read, explain & order whole numbers Explain negative numbers Explain & evaluate powers Non-Integers Explain meaning of digits & order numbers to 3 decimal places	Integers Recall & use addition/ subtraction/ multiplication facts Add, subtract, multiply, divide whole numbers Use & solve simple linear equations Use, sketch & interpret graphs Write & solve story & practical problems with whole numbers using any combination of operations Estimating Make & check sensible estimates Non-Integers Use, find & express fractions or percentages or decimals of a whole Write & solve story & practical problems with decimals, fractions Find & convert equivalent fractions-decimals-percentages using any combination of operations	Sequential & Repeating Patterns Continue, describe, find & make up rules for number & spatial patterns Use rules to predict patterns Number Properties Solve simple linear equations Knowledge of order of operation convention	Position Describe, draw, specify & interpret position with direction, distance, scale maps, bearings or grid references Metric Measure & estimate units of length, mass, volume, area, temperature, capacity Measure & read units & scales to nearest gradation Time Read & convert digital & analogue clocks Perform time calculation with 12 & 24 hour clocks Know units of time Read, interpret & construct time statements, scales, tables & charts	2D & 3D Shape Name & describe features of 2D & 3D objects Calculate perimeter, area, volume Describe symmetries Angle Identify clockwise, anticlockwise, quarter & half turns Know about simple angles including 90° (right-angle) & 180°, 30°, 45° & 60°	Symmetry & Transformations Create, describe & design geometric patterns with translation, reflection & rotation Enlarge or reduce 2D objects Angle Make turns Use protractor to measure angles Construct & Draw Make, model, construct, draw, name & describe 2D & 3D shapes Design & make containers or nets for simple polyhedrons	Trial Plan investigations & collate appropriate data Predict events by likelihood Model Compare, count & diagram possible outcomes Assign, predict probabilities & frequencies of events Estimate frequencies & mark on scale	Investigate Plan investigation & collect data Collect, display, data Choose & construct data displays Design & use simple scales Interpret Discuss & report distinctive features of data displays Make & evaluate statements about & interpretations of data

asTTle Mathematics Curriculum Map

Within the categories of the asTTle Mathematics Framework we considered all of the achievement objectives of the curriculum and organised them into 8 tables that attempt to show the progression of different mathematical topics (Appendix 1). The notation Xa-b in parenthesises behind achievement objectives indicates by 'X' the strand of the curriculum (A = Algebra; G = Geometry; M = Measurement; N = Number; and S = Statistics); by 'a' the level of the achievement objective (a = 2, 3, 4, 5 or 6); and by 'b' the number of the achievement objective as listed in order in the curriculum document. Hence, G2-4 is the achievement objective 'make clockwise and anticlockwise turns', which is the fourth geometry achievement objective listed for curriculum level 2 in the official mathematics curriculum statement.

The rows of the tables correspond to the key items of the Mathematics Framework. First the key themes are named, then the appropriate achievement objectives of the curriculum for Levels 2 to 6 are listed, and finally the Mathematics achievement standards for NCEA Level 1 are listed (NCEA, 2002). The NCEA Level 1 achievement standards are shown as a reference point and should not be considered as indicative of a next higher level. The relationship of NCEA Levels and grades to Mathematics Curriculum levels requires analysis beyond the scope of this report. From here the achievement objectives were drawn together into the 21 related themes that can be found in the revised Levels 2—6 asTTle Mathematics Framework (Table 2).

Recommended Changes to the asTTle Mathematics Framework & Curriculum Map

In evaluating the Mathematics Framework in the light of incorporating Levels 5 to 6, we felt that a number of changes were necessary. Before giving the suggested improvements, we should point out that none have a significant effect on the already completed Item Signature process for Levels 2-4, nor on the just completed asTTle V2 software (Ministry of Education & University of Auckland, 2003). An important but subtle change affects the naming of one strand, and one content area.

Number to Algebra

We have changed the title from Number & Algebra, to Number to Algebra. In Technical Report No.25 (Thomas et al, 2002), we indicated that there was a natural historical and mathematical development from Number to Algebra. Algebra is an extension of Number in that all of the basic rules of algebraic manipulation adopt the axioms of Number. But Algebra develops from Number and goes further than Number. By the time we reach Level 7, Algebra becomes increasingly significant. Indeed there are no Number achievement objectives above Level 6. We feel that the new title emphasizes this progression and development.

Algebra

To reflect the significance of Algebra at Levels 5 and 6 we have renamed the Patterns in Number content area as Algebra. We anticipate primary school users of asTTle will have little difficulty in understanding or adapting to this change.

Algebraic Manipulation

We have replaced Number Properties with a sub-heading called Algebraic Manipulation. This sub-heading includes the order of operations bullet point and we have added to it other aspects associated with the manipulation and solution of algebraic equations. The sub-heading reflects the increased focus on "real" algebra at Levels 5 and 6. Consequently we have also moved one of the bullet points from the Table 1 Whole Numbers sub-heading to this new Algebraic Manipulation. Note that movement of the achievement objectives across key themes within the asTTle tool will not affect the types of items selected for a test or the reporting of student achievement.

Table 2.
Mathematics Framework (Levels 2-6)

Number to Algebra			Measurement	Geometry		Statistics	
Number Knowledge	Number Operations	Algebra	Measurement	Geometric Knowledge	Geometric Operations	Probability	Statistics
Integers - Read, explain & order whole numbers - Explain negative numbers - Explain & evaluate powers **Non-Integers** - Explain meaning of digits & order numbers to 3 decimal places - Explain irrational numbers	**Integers** - Recall & use addition/ subtraction/ multiplication facts - Add, subtract, multiply, divide whole numbers - Write & solve story & practical problems with whole numbers using any combination of operations - Knowledge of order of operation convention - Estimating - Make & check sensible estimates **Non-Integers** - Use, find & express fractions or percentages or decimals of a whole - Write & solve story & practical problems with decimals, fractions - Find & convert equivalent fractions-decimals-percentages using any combination of operations	**Sequential & Repeating Patterns** - Continue, describe, find & make up rules for number & spatial patterns - Use rules to predict patterns **Algebraic Manipulation** - Knowledge of order of operation convention - Use and solve simple linear equations Simplify, factorise and expand algebraic expressions - Solve simultaneous equations **Graphs & Relations** - Use, sketch & interpret graphs - Graph linear, quadratic and exponential functions	**Position** - Describe, draw, specify & interpret position with direction, distance, scale maps, bearings or grid references - Metric Measure & estimate units of length, mass, volume, area, temperature, capacity - Measure & read units & scales to nearest gradation **Time** - Read & convert digital & analogue clocks - Perform time calculation with 12 & 24 hour clocks - Know units of time - Read, interpret & construct time statements, scales, tables & charts	**2D & 3D Shape** - Name & describe features of 2D & 3D objects - Calculate perimeter, area, volume - Describe symmetries **Angle** - Identify clockwise, anticlockwise, quarter & half turns - Know about simple angles including 90° (right-angle) & 180°, 30°, 45° & 60°	**Symmetry & Transformations** - Create, describe & design geometric patterns with translation, reflection & rotation - Enlarge or reduce 2D objects **Angle** -Make turns -Use protractor to measure angles **Construct & Draw** - Make, model, construct, draw, name & describe 2D & 3D shapes - Design & make containers or nets for simple polyhedrons	**Trial** - Plan investigations & collate appropriate data - Predict events by likelihood **Model** - Compare, count & diagram possible outcomes - Assign, predict probabilities & frequencies of events - Estimate frequencies & mark on scale	**Investigate** - Plan investigation & collect data - Collect, display, data - Choose & construct data displays - Design & use simple scales **Interpret** - Discuss & report distinctive features of data displays - Make & evaluate statements about & interpretations of data

Graphs and Relations

Currently the Level 2 to 4 relationship graphs are found under Whole Numbers. When considering this on the Level 2 to 6 continuum a more important feature starts to emerge. This is the fundamental, universal mathematical concept of a function. According to the rule of the function, *f*, for every value of *x*, there is a value of *f* called *f(x)*. The relations defined by functions are what are shown by a graph. The importance of graphs is the way that we can use them to visualize quite abstract relations. These relations are generalizations of the elementary patterns that are to be found up to about Level 4, where *x* is restricted to being a whole number. The relevance of the achievement objectives that are to be found in this part of the Curriculum Map is that they enable a function to be analysed to determine important aspects of its shape and to apply the function in some way. To emphasise the relation aspect of this collection of achievement objectives we propose the addition of a new sub-heading of Graphs & Relations to the Algebra content area.

Graphs also occur in the Statistics Strand. Their use is specifically to present data in that context. Although the conclusions drawn from these graphs are similar in many ways to that of the graphs in Graphs & Relations, their purpose is so specialized that it is best to leave them in the Statistics section of the Mathematics Framework.

Irrationals

At Levels 5 and 6 there are two achievement objectives that fit into no set that exists in the Level 2—4 Mathematics Framework of Table 1. These are N5-3 and N6-2. We have put them under a new sub-heading 'Irrationals' which becomes the third sub-heading under Number Knowledge.

N5-3 is about square roots. This is neither a part of Whole Numbers nor Fractions and Decimals. The new sub-heading of Irrationals is appropriate as square roots are mainly irrational numbers. N6-2 is about distinguishing between rational and irrational numbers and clearly this fits under the Irrationals heading. Again this does not fit under either Whole Numbers or Fractions & Decimals, but has been included in both as well as under the new sub-heading of Irrationals.

Missing Achievement Objectives

From work that has taken place relating to the mathematics section of the National Exemplar Project, and the Numeracy Project, we felt that certain key ideas were missing from the Achievement objectives in MiNZC (see www.nzmaths.co.nz). We list these below and note that they are numbered as EP-n in the Mathematics Curriculum Map:

- Make quarter and half turns (EP-1);
- Know about simple angles including 90º (right angle) and 180º; 30º, 45º and 60º (EP-2);
- Be able to use a protractor to measure angles to the nearest gradation (EP-3);
- Assign numerical probability values to simple events (EP-4);
- Use possible outcomes to assign probabilities (EP-5);
- Accurately describe aspects of the statistical situation represented by a statistical data display drawn by others (EP-6).

Implications for asTTle

A number of consequences arise from this curriculum mapping for the asTTle software. These consequences are largely to do with the size and number of achievement objectives available at the intersection of content and level in Levels 5 to 6 and are a related to the goal of providing valid and reliable measurement of performance.

For adequate estimation of performance each achievement objective requires a minimum of five items. The asTTle Mathematics curriculum map identifies a large number of objectives spread over eight significant content. This means that an item bank of some 1200 items for Levels 2 to 6 would still only provide relatively shallow coverage over the full range of achievement objectives in mathematics. A significantly larger number than the presently contracted 250 items per curriculum level would be needed in order to give rich representation to the complexity of the achievement objectives.

Furthermore, there are some areas of the curriculum that are expressed with one achievement objective, yet that objective represents many teaching units or points requiring significant teaching and learning time (e.g., find unknown angles and lengths in practical problems which can be modelled by triangles, using scale drawing, angle properties of triangles, Pythagoras' theorem, trigonometric ratios, the sine rule, or the cosine rule (G6-2)). This objective, were it to be measured by only five items, would be significantly and inappropriately reported on by asTTle. This suggests that a larger number of items are needed for some objectives compared to others. However, there is little data or direction at this point to indicate nation-wide curriculum emphases that might guide the development of unequal distribution of items by content.

A related, but inverse problem, has been encountered in trying to develop items for certain content areas that have a few and relatively 'thin' achievement objectives. One of the goals of asTTle is to have sufficient items for each content area at each curriculum level. Thus, approximately 35 items are required covering all the achievement objectives in each cell—content intersection. Normally, this is not a problem as most intersections have five or more objectives. However, there are two such intersections, which have very few achievement objectives; specifically, Levels 5 and 6 Number Knowledge and Measurement are represented by only one or two achievement objectives. Furthermore, the objectives in Measurement are those that are much more suited to practical activities than completion in paper and pencil environments. It is hoped that teachers of Levels 5 and 6 would not seek those kinds of assessments but a strategy to handle such requests will have to be developed.

One option would be to create many variant items around the limited objective of identifying irrational square roots, for example. This approach makes a relatively straightforward learning objective as important as the one cited above, and, more importantly, provides no further valid information once a student has demonstrated competence or incompetence five times. A second option would be to extend the teacher's selection of such an intersection by automatically including more items for that content or for that difficulty. This approach, while more sensible, would require extensive and clear explanation and training for teachers. A third option would be to deny teachers the power to create such tests by analysing the teacher's selection by the number of items potentially available and forcing the teacher to revise test selection criteria. This would be the most efficient approach but would require some software programming changes to the asTTle application.

The asTTle curriculum map distinguishes between knowledge and operations in the fields of Number and Geometry. In order to communicate clearly to teachers what students are succeeding or struggling in it was decided to reflect this orientation to achievement objective wording embedded in the asTTle reporting. By this, it is meant that an objective in the curriculum relating to geometry (e.g., make quarter and half turns) contains an element of knowledge and operations. As such, questions crafted around a number or geometry objective probably relate to either the knowledge or the operational aspect of the objective. To accurately report such measurement, the achievement objectives were reworded within the asTTle reporting to reflect such focus (e.g., know about and identify quarter and half turns; or make quarter and half turns). This approach, while assisting teachers in knowing accurately what learning intentions the items are measuring and reporting on, may be considered to be an inappropriate tampering with the curriculum.

It should also be noted that the actual wording of the achievement objectives in the curriculum map may have to be further modified to fit the constraints of the asTTle software (such as space available in the Individual Learning Pathways report). These wording changes may be seen by some as inappropriate treading on the curriculum stocktake's priority responsibility to revise or change the curriculum. Note that the wording in Appendix 1 is based on the curriculum statement rather than the implemented asTTle reporting.

A further issue to be considered in implementing Levels 5 and 6 in asTTle, is the relationship of achievement objectives to qualifications. It is apparent that some objectives in Level 6 may not be taught to students in Year 11 because those objectives are part of NCEA qualifications at Level 2. Those items, though legitimately part of Level 6, would not be welcome in a Level 6 test for Year 11 students, though they would be appreciated at Year 12. There are two implications from this: (a) the placement of those NCEA Level 2 but Curriculum Level 6 tasks in Year 12 trial papers to ensure maximum opportunity to learn, and (b) the need to more closely refine test creation in asTTle to permit exclusion of certain difficulty ranges or achievement objectives for use in certain situations. Changing the difficulty selection in asTTle from just a Level or Levels to sub-levels may work to resolve this latter difficulty. A graphic equaliser device calibrated to 5B, 5P, 5A, 6B, 6P, 6A may permit the exclusion of certain ranges within levels and some consideration of this by the asTTle team and the Ministry is justified.

Conclusion

The asTTle Mathematics Framework, used in developing assessments for Curriculum Levels 2—4, is readily adapted, with only a few changes to nomenclature or the addition of further achievement objectives or key themes, to the requirements of extending those assessments to Levels 5—6. The revised Mathematics Framework is an adequate breakdown of the big ideas of mathematics suitable for teachers' use in developing 40-minute paper-and-pencil classroom assessments of curriculum learning across Years 5 to 10 in New Zealand schools. Nevertheless, the Ministry of Education will need to give consideration to the implications and set direction and priorities in terms of curriculum and developmental priorities and costs for the asTTle software.

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Appendix 1. asTTle Mathematics Curriculum Map

Number Knowledge

	L2	L3	L5	L5	L6	NZEA
Integers	Read any 3-digit whole number (N2-1) Explain the meaning of the digits in 2- or 3-digit whole numbers (N2-2) Order any set of three or more whole numbers (up to 99) (N2-3) Represent a sum of money by two or more different combinations of notes and coins (M2-3)	Explain the meaning of the digits in any whole number (N3-1)	Explain the meaning of negative numbers (N4-1) Explain the meaning and evaluate powers of whole numbers (N4-2)	Convert numbers expressed in standard form to ordinary form, and vice versa (N5-1)	Classify numbers as whole, integer, rational, or irrational (N6-2)	
Non Integers	Write and solve story problems which involve halves, quarters, thirds, and fifths (N2-5)	Explain the meaning of the digits in decimal numbers with up to 3 decimal places (N3-2) Order decimals with up to three decimal places (N3-3)		Convert numbers expressed in standard form to ordinary form, and vice versa (N5-1)		
				Express the values of square roots in approximate and exact forms (N5-3)	Classify numbers as whole, integer, rational, or irrational (N6-2)	

Number Operations

	L2	L3	L4	L5	L6	NZEA
Integers	Mentally perform calculations involving addition and subtraction (N2-8) Recall the basic addition and subtraction facts (N2-7) Demonstrate the ability to use the multiplication facts (N2-9) Write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (N2-10) Write and solve comparison problems (N2-4) Write and solve story problems which require a choice of any combination of the four arithmetic operations (N2-11) Give change for sums of money (M2-2) Use the mathematical symbols =,<,> for the relationships "is equal to", "is less than", and "is greater than" (A2-3)	Recall the basic multiplication facts (N3-5) Solve practical problems which require finding fractions of whole number and decimal amounts (N3-7) Write and solve problems which involve whole numbers … and which require a choice of one or more of the four arithmetic operations (N3-6)	Explain satisfactory algorithms for addition, subtraction, and multiplication (N4-10) Demonstrate knowledge of the conventions for order of operations (N4-11)	Solve problems involving positive and negative numbers, using practical activities or models if needed (N5-7)	Devise a strategy to solve a number problem (N6-3)	AS90151Mathematics 1.7 Solve straightforward number problems in context
Non Integers	Write and solve story problems which involve halves, quarters, thirds, and fifths (N2-5)	Solve practical problems which require finding fractions of whole number and decimal amounts (N3-7) Write and solve problems which involve decimals and require a choice of one or more of the four arithmetic operations (N3-6)	Find a given fraction or percentage of a quantity (N4-9) Write and solve problems involving decimal multiplication and division (N4-8) Express quantities as fractions or percentages of a whole (N4-6) Find fractions equivalent to one given (N4-3) Express a fraction as a decimal, and vice versa (N4-4) Express a decimal as a percentage, and vice versa (N4-5)	Express one quantity as a percentage of another (N5-8) Solve problems involving positive and negative numbers, using practical activities or models if needed (N5-7) Increase and decrease quantities by given percentages, including mark up, discount, and GST (N5-9) Share quantities in given ratios (N5-10) Solve practical problems involving decimals and percentages (N5-6) Express the values of square roots in approximate and exact forms (N5-3)	Devise a strategy to solve a number problem (N6-3) Perform basic operations on fractions and mixed numbers (N6-1)	AS90151Mathematics 1.7 Solve straightforward number problems in context
Estimating (e)	Make sensible estimates and check the reasonableness of answers (N2-6)	Make sensible estimates and check the reasonableness of answers (N3-4)	Make sensible estimates and check the reasonableness of answers (N4-7)	Make sensible estimates and check the reasonableness of results (N5-4) Round numbers sensibly (N5-2)	Estimate and calculate answers, making efficient use of a calculator, where appropriate, as part of solving a problem (N6-4) Discuss the reasonableness and meaning of the answers obtained in solving a problem (N6-5)	

Algebra

	L2	L3	L4	L5	L6	NZEA
Algebraic manipulation		Solve problems of the type □+*15* = *39* (A3-5)	Solve simple linear equations such as *2* □+ *4* = *16* (A4-5) Demonstrate knowledge of the conventions for order of operations (N4-11)	Combine like terms in algebraic expressions (A5-7) Factorise and expand algebraic expressions (A5-9) Simplify algebraic fractions (A5-8) Use equations to represent practical situations (A5-10) Evaluate linear expressions by substitution (A5-5) Solve linear equations (A5-6)	Substitute values into formulae (A6-6) Form and solve linear equations, simultaneous equations, and simple quadratic equations (A6-5) Generate linear and quadratic patterns and find and justify the rule (A6-2)	AS90147 Mathematics 1.1 Use straightforward algebraic methods and solve equations
Sequential and repeating pattern(s)	Continue a sequential pattern and describe a rule for this (A2-1)	Describe in words, rules for continuing number and spatial sequential patterns (A3-1) Make up and use a rule to create a sequential pattern (A3-2) State the general rule for a set of similar practical problems (A3-3)	Find a rule to describe any member of a number sequence and express it in words (A4-1) Use a rule to make predictions (A4-2) Find and justify a word formula which represents a given practical situation (A4-4)	Generate patterns from a structured situation, find a rule for the general term, and express it in words and symbols (A5-1) Generate a pattern from a rule (A5-2)	Interpret and use information about rates presented in a variety of ways, for example, graphically, or in tables (M6-4) Generate a pattern from a rule (A6-3)	
Graphs and relations	Use graphs to illustrate relationships (A2-2)	Use graphs to represent number, or informal, relations (A3-4)	Sketch and interpret graphs on whole number grids which represent simple everyday situations (A4-3)	Sketch and interpret graphs which represent everyday situations (A5-3) Graph linear rules and interpret the slope and intercepts on an integer co-ordinate system (A5-4) Interpret and use information about rates presented in a variety of ways, for example, graphically, numerically, or in tables (M5-3)	Form and interpret a graph (A6-1) Explain the relationship between the gradient of a graph and the rate of change (M6-3) Design and use a 2-dimensional scale to represent data (M6-2) Graph linear, quadratic, and exponential functions, and relations of the form $x^2+y^2=r^2$ and xy = c (A6-4)	AS90148Mathematics 1.2 Sketch and interpret linear or quadratic graphs

Statistics

	L2	L3	L4	L5	L6	NZEA
Investigate and Display (d)	Collect and display category data and whole number data in pictograms, tally charts, and bar charts, as appropriate (S2-1)	Collect and display discrete numeric data in stem-and-leaf graphs, dot plots, and strip graphs, as appropriate (S3-2)	Choose and construct quality data displays (frequency tables, bar charts, and histograms) to communicate significant features in measurement data (S4-3) Collect and display time-series data (S4-4)	Plan and conduct statistical investigations of variables associated with different categories within a data set, or variations of variables over time (S5-1) Consider the variables of interest, identify the one(s) to be studied, and select and justify samples for collection (S5-2)	Formulate statistical questions about situations involving possible relationships between variables (S6-1) Formulate questions about variations over time in continuous processes (S6-2)	AS90193Mathematics 1.5 Use straightforward statistical methods to explore data
			Design and use a simple scale to measure qualitative data (M4-4)	Find, and authenticate by reference to appropriate displays, data measures such as mean, median, mode, inter-quartile range, and range (S5-3)	Collect bi-variate measurement and discrete number data, and clearly and concisely communicate the significant features in appropriate displays, including scatter plots (S6-4)	
		Plan a statistical investigation of an assertion about a situation (S3-1)	Plan a statistical investigation arising from the consideration of an issue or an experiment of interest (S4-1)	Collect and display comparative samples in appropriate displays such as back-to-back stem-and-leaf, box-and-whisker, and composite bar graphs (S5-5)	Make and justify statements about relationships between variables in a sample as a result of a statistical investigation (S6-5) Identify long-term and short-term features in time-series data (S6-6)	
			Collect appropriate data (S4-2)		Identify data collection methodology (S6-3)	
Interpret (i)	Talk about the features of their own data displays (S2-2)	Use their own language to talk about the distinctive features, such as outliers and clusters, in their own and others' data displays (S3-3)	Report the distinctive features (outliers, clusters, and shape of data distribution) of data displays (S4-5)	Discuss discrete and continuous numeric data presented in quality displays (S5-4)	Suggest improvements in the investigation, design, data collection, or display, where possible inferences are inconclusive (S6-7)	
	Make sensible statements about the situation represented by a statistical data display drawn by others (S2-3)	Make sensible statements about an assertion on the basis of the evidence of a statistical investigation (S3-4)	Evaluate others' interpretations of data displays (S4-6)	Use data displays and measures to compare data associated with different categories (S5-6)		
		Accurately describe aspects of the situation represented by a statistical data display drawn by others (EP-6)	Make statements about implications and possible actions consistent with the results of a statistical investigation (S4-7)	Make statements about time-related variation as a result of a statistical investigation (S5-7)		
				Report on possible sources of error and limitations of an investigation (S5-8)		

Probability

	L2	L3	L4	L5	L6	NZEA
Trial (t)		Plan a statistical investigation of an assertion about a situation (S3-1) Predict the likelihood of outcomes on the basis of a set of observations (S3-6)	Collect appropriate data (S4-2)	Determine probabilities of events based on observations of long-run relative frequency (S5-9) Predict the outcome of a simple probability experiment, test it, and explain the results (S5-11)		
Model (m)	Compare familiar or imaginary, but related, events and order them on a scale from least likely to most likely (S2-4)	Use a systematic approach to count a set of possible outcomes (S3-5) Assign numerical probability values to simple events (EP-4) Predict the likelihood of outcomes on the basis of a set of observations (S3-6)	Find all possible outcomes for a sequence of events, using tree diagrams (S4-9) Use possible outcomes to assign probabilities (EP-5) Estimate the relative frequencies of events and mark them on a scale (S4-8)	Determine the theoretical probabilities of the outcomes of an event such as the rolling of a die or drawing a card from a deck (S5-10) Find the probability of a given sequence of events, using tree diagrams (S5-12)	Use tables of multi-variate data from social contexts to find the probabilities of everyday events or the proportion of outcomes in a given category (S6-8) Determine the theoretical probabilities of the outcomes of both exclusive and independent events such as the rolling of a die followed by the drawing of a card from a deck (S6-9) Use probability trees to calculate conditional probabilities (S6-10)	AS90194Mathematics 1.6 Calculate relative frequencies and theoretical probabilities

Geometric Knowledge

	L2	L3	L4	L5	L6	NZEA
Angle (a)	Make clockwise and anticlockwise turns (G2-4) Make quarter and half turns (EP-1)	Know about simple angles including 90° (right-angle) and 180°; 30°, 45° and 60° (EP-2)	Be able to use a protractor to measure angles to the nearest gradation (EP-3)	Use the angle properties of parallel lines and explain the reasoning involved (G5-1) Apply the symmetry and angle properties of polygons (G5-2 Use the angle between a tangent and radius property, and the angle-in-a-semicircle property (G5-3) Find an unknown side in a right-angled triangle, using scale drawing, Pythagoras' theorem, or an appropriate trigonometric ratio (G5-5)	Identify and solve right-angled triangles within 3-dimensional objects and drawings (G6-3) Find unknown angles and lengths in practical problems which can be modelled by triangles, using scale drawing, angle properties of triangles, Pythagoras' theorem, trigonometric ratios, the sine rule, or the cosine rule (G6-2)	AS90152Mathematics 1.8 Solve right-angled triangle problems
Two dimensional and three dimensional shapes (s)	Make, name, and describe, using their own language and the language of geometry, everyday shapes and objects (G2-1)	Describe the features of 2-dimensional and 3-dimensional objects, using the language of geometry (G3-1) Describe patterns in terms of reflection and rotational symmetry, and translations (G3-6)	Calculate perimeters of circles, rectangles, and triangles, areas of rectangles, and volumes of cuboids from measurements of length (M4-2) Describe the reflection or rotational symmetry of a figure or object (G4-7)	Find perimeters, areas, and volumes of everyday objects (including irregular and composite shapes), and state the precision (limits) of the answer (M5-1) Recognise when 2 shapes are similar, find the scale factor, and use this to find an unknown dimension (G5-9)	Explore and describe a locus formed in a practical situation (G6-4)	
	Make, name, and describe, using their own language and the language of geometry, everyday shapes and objects (G2-1)	Describe the features of 2-dimensional and 3-dimensional objects, using the language of geometry (G3-1)	Calculate perimeters of circles, rectangles, and triangles, areas of rectangles, and volumes of cuboids from measurements of length (M4-2) Describe the reflection or rotational symmetry of a figure or object (G4-7)	Find perimeters, areas, and volumes of everyday objects (including irregular and composite shapes), and state the precision (limits) of the answer (M5-1)	Identify and solve right-angled triangles within 3-dimensional objects and drawings (G6-3)	

Geometric Operations

	L2	L3	L4	L5	L6	NZEA
Angle (a)	Make clockwise and anticlockwise turns (G2-4) Make quarter and half turns (EP-1)		Be able to use a protractor to measure angles to the nearest gradation (EP-3)	Use the symmetry and angle properties of polygons to solve practical problems (G5-10)		
Symmetry and Transformations (s)	Create and talk about geometric patterns which repeat (show translation), or which have rotational or reflection symmetry (G2-3)	Describe patterns in terms of reflection and rotational symmetry, and translations (G3-6)	Describe the reflection or rotational symmetry of a figure or object (G4-7)	Solve practical problems which can be modelled, using vectors (G5-8)	Apply the relationship between the scale factors for length, area, and volume (G6-5)	AS90153Mathematics 1.9 Use geometric reasoning to solve problems
		Design and make a pattern which involves translation, reflection, or rotation (G3-7)		Recognise when 2 shapes are similar, find the scale factor, and use this to find an unknown dimension (G5-9)	Explain the effect of negative scale factors for enlargement (G6-6)	
			Apply the symmetries of regular polygons (G4-6)	Identify and use invariant properties under transformations (G5-12)	Describe the net effect of combining two or more transformations (reflections, rotations, or translations) (G6-7)	
		Enlarge, on grid paper, simple shapes to a specified scale (G3-8)	Enlarge and reduce a 2-dimensional shape and identify the invariant properties (G4-8)	Use and interpret vectors which describe translations (G5-11)		
Construction and drawing (c)	Make, name, and describe, using their own language and the language of geometry, everyday shapes and objects (G2-1)		Make a model of a solid object from diagrams which show views from the top, front, side, and back (G4-3)	Construct right angles, parallel and perpendicular lines, circles, simple polygons, medians, mediators, altitudes, and angle bisectors (G5-4)		AS90150Mathematics 1.4 Use geometric techniques to produce a pattern or object
		Model and describe 3-dimensional objects illustrated by diagrams or pictures (G3-3)	Construct triangles and circles, using appropriate drawing instruments (G4-1)			
		Draw pictures of simple 3-dimensional objects (G3-4)	Draw diagrams of solid objects made from cubes (G4-4)	Make isometric drawings of 3-dimensional objects built out of blocks (G5-6)	Draw and interpret 2-dimensional representations of 3-dimensional objects (G6-1)	
		Design and make containers to specified requirements (G3-2)	Design the net and make a simple polyhedron to specified dimensions (G4-2)			

Measurement (M)

	L2	L3	L4	L5	L6	NZEA
Time (t)	Read time and know the units of time & minute, hour, day, week, month, and year (M2-4)	Read and interpret everyday statements involving time (M3-3) Show analogue time as digital time, and vice versa (M3-4)	Perform calculations with time, including 24-hour clock times (M4-5) Read and construct a variety of scales, timetables, and charts (M4-3)			
Position (p)	Describe and interpret position, using the language of direction and distance (G2-2)	Draw and interpret simple scale maps (G3-5)	Specify location, using bearings or grid references (G4-5)			
Metric measurement (m)	Carry out practical measuring tasks, using appropriate metric units for length, mass, and capacity (M2-1)	Demonstrate knowledge of the basic units of length, mass, area, volume (capacity), and temperature by making reasonable estimates (M3-1) Perform measuring tasks, using a range of units and scales (M3-2)	Carry out measuring tasks involving reading scales to the nearest gradation (M4-1)	Design and use models to solve measuring problems in practical contexts (M5-2)	Demonstrate the knowledge and skills necessary to plan, implement, and evaluate practical measuring tasks (M6-1)	AS90149Mathematics 1.3 Solve problems involving measurement of everyday objects