Paper & Pencil Strategy Assessment Items for the New Zealand Number Framework: Pilot Study

Technical Report 18, Project asTTle, University of Auckland

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This report summarises results of a pilot study, using student focus groups (Years 5 to 7), of paper and pencil strategy assessment items that had been written to elicit information about student use of mental strategies as defined by the New Zealand Number Framework. Significant difficulties were encountered in mapping both items and responses to the Number Framework; however, about half of the items seemed to be effective in providing information from students as to the mental strategy they had used in answering the questions. Recommendations are made to guide future item development.

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Purpose

The aims of the pilot study were to determine (a) what strategies students in Years 5 to 7 used while solving paper and pencil mathematics problems and, (b) whether those strategies could be mapped back to the New Zealand Number Framework.

The specific objectives of this pilot study were to (a) identify whether student's responses, both oral and/or written, could be related back to the New Zealand Number Framework; (b) determine which mathematics problems do or do not elicit information about strategies the students used while solving them, and (c) recommend changes to the paper and pencil assessment items to improve their capacity to elicit information about strategies used.

Method

Subjects

Students, teacher nominated, from several Auckland schools participated in the study. The selected contributing schools, full primary schools, and two intermediate schools included the range of low-, medium- and high-decile schools. Participants included: 9 students from Year 5, 9 students from Year 6, and 9 students from Year 7. There were 27 (male n=13, female n=14) participants in total. Of these 3 were Maori, 1 of Pacific Island nations descent, 2 were of Asian descent, 3 were Indian, 2 were of Middle Eastern descent, and the remaining 16 were Pakeha/European.

Procedure

Information was gathered using focus groups. For each session (approximately one and a half hours), three students, from the same grade level, were presented with a set of mathematics questions that they first read and then answered in written form. After completion of a problem, each student was asked to verbalise how they had answered the problem, what approach they used and, if possible, why they selected that approach. Questions were asked if further elaboration was required. All responses were recorded, as accurately as possible, by the researcher. (See Appendix B for all student responses.) Note that identities of students are obscured by use of initials.

Materials

Items were written at a workshop intended to draft paper and pencil mathematics assessment items that would elicit student strategies and processes according to the New Zealand Number Framework. Participants included 7 practicing teachers who were trained in the New Zealand Number Framework and one of the New Zealand Number Framework designers. 39 items were selected for piloting and then typed, copied and assembled into booklets of one item per page (Appendix A).

Summary of Findings

The New Zealand Number Framework (NZNF) is comprised of seven strategy stages (Table 1). Three strategy stages (pre-counting, count all from one and advanced counting) are considered part of the global strategy stage of "counting." Four strategy stages (early additive, advanced additive, advanced multiplicative, and advanced proportional) as considered part of the global strategy stage of "part/whole." Each stage contains the operational domains of addition and subtraction, place value, multiplication, and factions. Listed within each strategy stage and operational domain are several key strategies that a child may use to solve problems. Key strategies, for example at the global stage five (advanced additive/early multiplicative) in the operational domain of addition/subtraction may be (a) making tens, (b) compensation, and (c) reversibility.

Table 1 shows that 37 of the 39 items could, to some degree, be mapped to the Number Framework. Eleven of the items could be mapped with some certainty but only three of those provided some evidence in written student responses that mapped back to the framework (i.e., items 7, 22, & 26). 26 items could not be mapped with much certainty; however, they did seem to provide some evidence, especially in terms of the oral explanations given by students, of strategies that could be mapped to the Number framework. Although there may be evidence of written and/or oral strategies, this was not necessarily across all students. Furthermore, not all responses were consistent. For example, there may have been a number of students who gave oral and written explanations that matched, but some student's oral responses did not always match written responses. Furthermore, the strategies noted were not always clearly explained.

The findings from this study will be discussed in two parts: First, whether the problems can be mapped back to strategy stage and/or operational domain of the NZNF. Second, whether the problems elicited use of key strategies by the student as they solved them.

Relating Problems Back to the Strategy Stage/Operational Domain

As a part of the analysis of the data collected, the questions were analysed to ascertain whether they related back to the NZNF in terms of strategy stage and/or operational domain (Table 1). It was possible to identify what strategy stage and/or operational domain most problems represented and, subsequently, the strategy stage the student may be working at if based on a correct or incorrect response. For example, item 29 involved adding and subtracting multi-digit numbers that, if answered correctly by the student, would suggest that the student is working at an advanced additive strategy stage in the operational domain of place value. There are, however, several issues and concerns that need to be noted.

First, there were difficulties of the assigned strategy stage and/or operational domain to the problems. Recorded above several problems were the strategy stage and/or operational domain they were designed for. For some problems, however, the strategy stage identified appears incorrect and/or confusing. Item 27, for example, the strategy stage identified for the problem is early multiplicative yet the problem involves working with multi-digit numbers. Multiplying multi-digit numbers is at the advanced multiplicative strategy stage. In item 11 only the word "advanced" is typed at the top of the problem. It is unclear whether the problem is best suited for the advanced additive/early multiplicative or advanced multiplicative stage. The question itself involved multiplication, addition and subtraction of single and multi-digit numbers. A number of problems did not identify what strategy stage and/or operational domain they were designed for and also appear confusing as to what strategy stage they represent. Items 28 and 36 highlight this issue.

Table 1

Mathematical Problems as Related to New Zealand Number Framework

Item	Can't Map	Pre-Counting	Count All From One	Advanced Counting				Early Additive				Advanced Additive / Early Multiplicative				Advanced Multiplicative / Early Proportional		Advanced Proportional
				Counting on	Early concept 10	Skip counting	Fraction by equal sharing	Early +/-	Early place value	Mult. by repeated +	Fraction of a no. by +	Ad. +/-	Ad. Place value	Early x	Fraction of a no. by mult. with +	Ad. Mult. / division	Early proportion	Advanced proportions
1																	*o	
2													?			*o		
3													*wo					
4													?wo					
5													?wo					
6																?o		
7																*wo		
8									?wo									
9									?wo									
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12							?											
13													?wo^					
14	*												?o					
15													*o					
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22									*wo									
23								?wo										
24																	?o	

Table continued

@ Written and oral explanations did match only in situations where, if able, they were able to copy the steps.

^ Note that some students were not able to use the number lines but were able to solve the equations either using some key strategy or a vertical algorithm.

Item	Can't Map	Pre-Counting	Count All From One	Advanced Counting					Early Additive			Advanced Additive / Early Multiplicative				Advanced Multiplicative / Early Proportional		Advanced Proportional
				Counting on	Early concept 10	Skip counting	Fraction by equal sharing	Early +/-	Early place value	Mult. by repeated +	Fraction of a no. by +	Ad. +/-	Ad. Place value	Early x	Fraction of a no. by mult. with +	Ad. Mult. / division	Early proportion	Advanced proportions
25														?ow&				
26																*wo		
27																?o		
28													?o					
29													*o					
30									*o									
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32								*o										
33								?o										
34									?o#									
35														?				
36	* *																	
37							?o											
38												?o						
39									?o!									

& Written explanations matched oral explanations to degree in that they had to write why they chose the particular clue.
$ Written form – majority wrote problem out in form of vertical algorithm.
# Majority written explanation – vertical algorithms also for second part to problem.
! This problem required the children to identify what the student had done wrong – some students were able to partition the problem but not work through what Kiah did wrong after that.

KEY:

? Can not fit problem(s) back to framework with certainty as could be placed in another strategy stage and/or operational domain but appears may fit here.

***** Can map problem(s) back to framework, based on reading of problem alone with certainty.

***w** Can map problem(s) back to framework / some evidence in the written explanation in terms of noted key strategies used that maps back to framework.

***o** Can map problem(s) back to framework / some evidence in the oral explanation in terms of noted key strategies used that maps back to framework.

***wo** Can map problem(s) back to framework / some evidence in both oral and written explanation that maps back to framework.

?w Can not map problem(s) back to framework with certainty / some evidence in the written explanation in terms of noted key strategies used that maps back to framework.

?o Can not map problem(s) back to framework with certainty / some evidence in the oral explanation in terms of noted key strategies used that maps back to framework.

?wo Can not map problem(s) back to framework with certainty / some evidence in both oral and written explanation that maps back to framework.

NOTE:

1. Where problem(s) have been mapped back to the framework placed in column that represents highest strategy this problem represents. For example, if question was designed to distinguish students between one strategy stage or another, highest strategy stage chosen.
2. Some problems have more then one part. Each part has been assessed independently where appropriate.
3. Some problems involved combinations of multiplication and/or division and/or addition and/or subtraction.

Second, there was difficulty relating the student's response back to the NZNF with answers that were poorly articulated. Item 28 required the student to identify an error made by Peter and explain why he did or did not get the correct answer. If the student could solve 61 – 29 this would suggest that the student is working at the early additive strategy stage. If, however, the student failed to provide an explanation as to what Peter did wrong, did that mean the student was not working at the early additive strategy stage? Item 36 required the student to determine who had the most bananas and then explain their answer. While all students were able to determine who had the most bananas suggesting that they are working at the early additive strategy stage, often their explanations were unsatisfactory. How this relates back to what strategy stage the student is at, again, is unclear.

Third, in addition to the articulation issue just highlighted, a number of problems required the student to give a written explanation as to how to solve the problem or what was wrong with someone else's working. Although many students could solve the equations presented in the problem, typically using vertical algorithms, they often failed to give a reasonable explanations as to what was incorrect with the other student's working and/or provide adequate explanations as to how to solve the problem. Furthermore, several students could only give oral explanations. Items 1, 2, 14, 19, 24, and 39 were typical of this difficulty.

Fourth, not all problems resulted in written responses that identified the strategy stage a child may be working at. Items 32 and 33 above all demonstrated this problem. These questions were designed to distinguish between counters and groupers yet this was difficult to establish from a circled answer of an equation. If the student got the answer correct, it is not possible to determine whether they answered it by using a counting or grouping strategy. Thus, inference about whether the student was working at an advance counting stage or early additive stage, was not possible to work out.

Fifth, a number of problems involved two or more sections to be answered. If a child failed to answer one section, it became unclear how to relate that failure back to a strategy stage. For example, in item 29 the students were first to determine how many seats in a movie theatre were empty. The majority of students were able to correctly answer this section to the problem. The students were then required to solve the problem in another way. Only one student (Year 7) was able to come up with a reasonable suggestion. Did the failure of the majority of students to answer the second section of the problem mean that they were now working at a lower strategy stage? Answers by students on items 14, 20, and 39 were further emphasized this issue.

Relating Problems Back to Key Strategies Used

The responses, both oral and written, that the students gave to the problems were examined to determine whether the problems elicited information about which key strategies were used. Items 3, 4, 5, 6, 8, 10, 13, 22, 23, 24, 25, 27, 34, and 39 did elicit examples, both in oral and written form, of several key strategies utilized. Strategies that were captured included compensation, counting on, place value partitioning, making tens, doubling, reversibility, inverse operations, and rounding. Interestingly, these problems typically listed strategies from which the children were to choose one that best represented how they solved the problem or asked the children to show the working they used to solve the problem. Several caveats, however, need to be noted.

First, limitations resulting from restricted option choice. Not all strategy choices as listed in problems represented the student's preferred strategy choice. Item 23 asked the students to choose between one of two possible strategies to solve addition problems. One strategy involved counting-on, a strategy if used would seem to place the student at the advanced counting stage. The other strategy involved making tens, which, if used, would suggest that the student is working at the early additive stage. Several students chose the counting on strategy as the way they solved the problems as they were confused by the other strategy. A number of students, however, stated that even though they chose the counting-on method, it was not their preferred method. They stated that they would have used another strategy such as a vertical algorithm. Therefore, identifying exactly what strategy stage these students are at is problematic as they may be beyond the global stage of advanced counting yet their answer suggested otherwise. Item 13 further delineated this issue. There were students who found it difficult using the number line to solve addition equations. This did not mean that the students could not solve the equations. They solved the problems using other strategies and then attempted to use the number line. Interestingly, another issue item 13 demonstrated was the limitations of prior teaching. The poor performance of a number of students in using the number line may be due to lack of instruction in number line use to solve addition problems.

Second, there is the quandary of a student's understanding and conception of a task influencing their answer. Several problems, for example items 3, 4, 22, 25, offer choices as to what key strategy the student may have used while solving the problem. All students were able to make a choice. However, does choice of one strategy accurately represent the strategy stage they are working at? Item 25 best highlighted this dilemma. The problem stated that the answer to 4 x 5 is unknown. The student is to select a clue that would be most helpful to solve that equation. Several students choose the clues "5, 10, 15" or "5 + 5 + 5" as they felt this would help the student solve the problem given a lack of knowledge of the times tables. The student would only have to add on one more lot of 5. Several other students, however, chose the clue "3 x 5" or "5 x 5" because those equations were closest to 4 x 5. The student could then either add on or subtract 5 or, instead, work their way up or down the times tables until they got to 4 x 5. If a student selected "5, 10, 15" this may misrepresent the strategy stage a student is actually able to work at. Furthermore, the majority of responses the students gave for their choice were reasonable.

Several problems failed to elicit much of any information as to what key strategies the students were using across the years. These problems fall into several groups. First, items 1, 2, 11, 14, 28, 31, 36, 39 required the student to identify errors made by others in their calculations, or provide proof for "logic" type problem. Orally most students were able to identify the errors or explain how they solved the "logic" type problems but as presently constructed the items elicited little written evidence beyond vertical algorithms. For example, in item 39 Kiah had incorrectly worked out 27 + 23 to equal 410. The students were to explain what they thought Kiah did wrong. A number of students were able to solve the problem correctly (27 + 23

= 50) but students, particularly in Years 6 and 7, were unable to come up with an explanation as to what Kiah did wrong. Other than gaining information as to whether or not the students were able to solve addition problems involving tens, little or no information is provided as to what key strategies the students used. Additionally, this re-emphasises concern related to articulation of response. If a student is unable to provide a satisfactory written explanation, does this mean that the student is not working at the strategy stage the question was designed for?

Second, for a number of students, particularly students in Year 7, a number of problems were "of little challenge." When asked how they solved the problems, many responded that they just "knew it" or it was a "basic fact that they had learned off by heart." Examples of problems that were of little challenge can be found on items 8, 9, 12, 22, 32, and 33.

Third, as already alluded to, a number of problems failed to elicit much, if any, information in written form as to what key strategies the students used which they were able to provide information in oral form. Items 15 – 18, 32, and 33 are exemplary examples of this. Items 15 to 18, for example, required students to check if addition equations were right or wrong and to write down what numbers the student added first. One student for the equation 62 + 120 + 137 = 320 wrote down that the first two numbers he used to solve the problem were 62 and 120. This provided little information about the key strategies he actually used. From the discussion it was found that the student was using a key strategy of standard partitioning as he added 62 and 120 by going 120 + 60 = 180, 180 + 2 – 182.

Finally, on a more positive note, several problems failed to elicit what key strategies students used not because of the problem type but because of their difficulty. Students in Year 5 had great difficulty in answering items 6, 7, 11, 20 (second part), 24, 26, and 27.

Recommendations

Whether it is possible to develop questions that identify what strategy stage a student is at and what key strategies the student used is difficult to determine. While it appeared that some problems might have worked, several issues and concerns were also identified. Based on this summary, several recommendations are made and attention is drawn to other concerns. This list is by no means exhaustive but may be considered a starting point for continued development and refinement of paper and pencil mathematics strategy problems.

- Clarify exactly what strategy stage and/or operational domain each question represents. This will enable better precision in determining what strategy stage(s) within a given operational domain a student is working at.
- Ensure that all strategy stages and operational domains are represented in the development of problems. For example, not all the strategy stages in the operational domain of fractions were represented.
- Develop questions that are difficult enough for higher achieving students that will "force" them to demonstrate/write out their working thus provide information as to what key strategies they are using.
- Develop a rubric for evaluating written explanations of problems.[1]

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1 Several articles that may be of use in developing rubrics and assessing student performance:

5. Further explore the use of questions that either list choice a of strategies and/or require the student to show their working.

Clarify how the use of a key strategy does or does not represent a strategy level a student is working at. Furthermore, clarify whether the use of one strategy over another is necessarily better or worse than the use of another strategy.

Give consideration to students with learning difficulties. For example, T, Year 7 student, had poor handwriting. As a result his written responses were short, difficult to read, and inadequate. They were not reflective of his actual knowledge as demonstrated by his oral responses.

Cai, J., Magone, M. E., Wang, N., & Lane, S. (1996). Describing student performance qualitatively. Mathematics Teaching in the Middle School, 1(10), 828-835.

Kenny, P. A., Zawojewski, J. S., & Silver, E. A. (1998). Marcy's dot pattern. Mathematics Teaching in the Middle School, 3(7), 474-477.

Lane, S. (1993). The conceptual framework for the development of a mathematics performance assessment instrument. Educational Measurement: Issues and Practice, 12(2), 16-23.

Appendices

Appendix A: Strategy Assessment Items

ITEM 1
Advanced multiplicative
Advanced multiplicative

Elena spends ½ of her money. Paul spends ¼ of his money.

But Paul spends more money than Elena.

How can this be true?

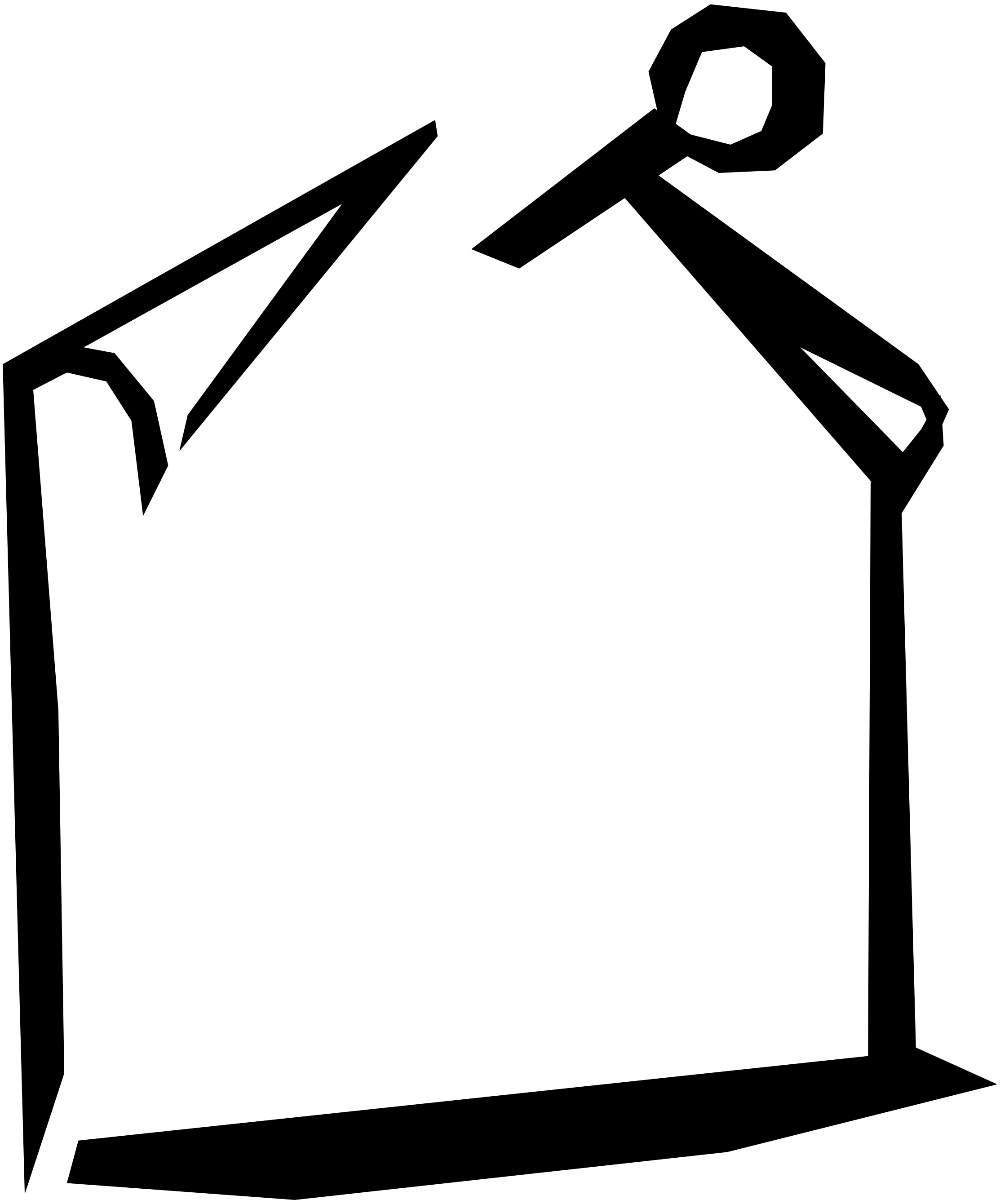
ITEM 2

Gavin is going to work out 3 x 2997.

Sione is going to work out 2996 + 2998 + 3000.

Sally looks at the numbers and without working out says, “I know Sione’s answer is 3 bigger than Gavin’s answer.”

Explain how Sally knows this.



ITEM 3

Advanced additive

Put the answer to this problem in the box.

399+601= $\boxed{}$

What was the first thing you did to work out this problem?

I wrote down the numbers one above the other.

I added the 3 and the 6 together.

I did another way. This is what I did……
In my mind I took one off the 601 and put it on the 399.
I added the 99 and the 1 together.

ITEM 4

Advanced additive

The teacher shows the class a pile of 79 sweets and another pile of 6 sweets.

She asks the class to work out the total number of sweets.

Fiona counted in her head like this:
80, 81, 82, 83, 84, 85

Peter said in his mind:
79 and 1 make 80.
There are 5 more to add on so the answer is 85.

Jemima writes down

$$\begin{array}{r} 79 \\ +\ 6 \\ \hline 85 \end{array}$$

Which method would you use?

Fiona's Peter's Jemima's

Another way.

ITEM 5
Advanced additive

Put the answer to this problem in the box:

$$51+49=\boxed{}$$

What was the first step you took to get the answer?

Nine plus one equals ten

Turned it into 50+50

Four and five is nine

Started to count from 52……

ITEM 6
N L4-7

The orchard report says 24.8% of 2000 apples are rotten.

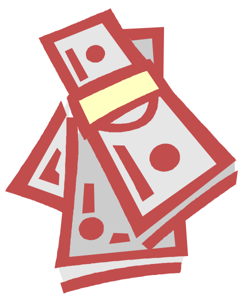
Henry says that means 81 apples must be rotten.

Larnie says this can’t be right.

How does Larnie know this?

ITEM 7
Advanced multiplicative

Marta is working out 15% of $80. She did two calculations.



One answer was $8 and the other answer was $4.

She added them to get $12. She said 15% of $80 was $12,

and she was right.

What did she do?

ITEM 8
Advanced counting/part-whole

Solve this problem:

13-4= $\boxed{}$

When you solved this, what did you do?

I went 12, 11, 10, 9 in my mind. The answer is 9.

I went 13, 12, 11, 10 in my mind. Nine were left.

I used my fingers and went 12, 11, 10, 9, 9 is the answer.

I took 3 from thirteen, leaving 10. I took one more away to make 9. The answer is 9.

13 is the same as 10 and 3. I took 4 from 10, leaving 6. Then I added the 3 to the 6, the answer is 9.

ITEM 9

9+ $\boxed{}$ =13

Peter has 9 sweets and he is given some more. Now he has 13.

Work out how many more he was given.

Which of these is like what you did?

I went 9 and 1 more makes 10. 10 and 3 more works 13. 1 and 3 is 4.

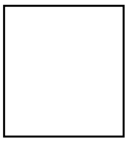
I went 10, 11, 12, 13 in my mind and said the answer is four.

I went 10, 11, 12, 13 on my fingers and said it was 4.

I didn't do it any of these ways. I did____________________

ITEM 10
Early additive

Write the answer to this problem in the box.

$29 + 5 =$ 

Tick the box that shows the way you worked it out?

I went 30, 31, 32, 33, 34 in my mind.

I went 30, 31, 32, 33, 34 on my fingers.

I went 29 and 1 is 30 and 4 more is 34.

A different way. I did ____________________

ITEM 11
Advanced
A toy costs $14.95.

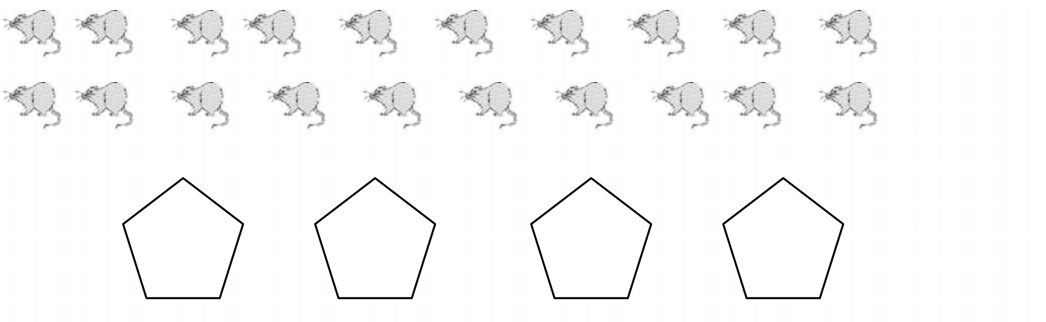
Grandma has to buy 6.
She quickly worked out in her head it would cost $89.70.
She did this by going:
$14.95 is close to $15.
$15 times 6 is $90.
$14.95 is 5 cents less than $15.
5 cents times 6 is 30 cents.
$90 take away 30 cents is $89.70.

The next day she went to buy 8 bottles of drink at $1.95 each. Explain how she quickly worked out the cost.

ITEM 12

Advance Counting (Multiplication – repeated addition)

Proficient



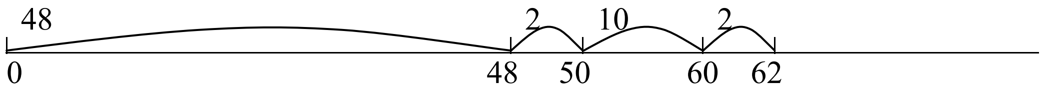
There are 20 mice and 4 mouse holes. A cat comes. The same number of mice goes into each hole. How would you solve this problem?

ITEM 13
Advance Additive

Proficient

Papa worked out on his number line.

$48 + \boxed{} = 62$ $2 + 10 + 2 = 14$



Now can you use the number line to work out these problems?

$66+ \boxed{} = 82$

$197+ \boxed{} =204$

$1789+ \boxed{} =1812$

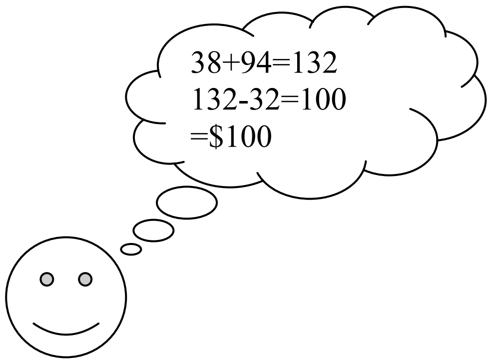
ITEM 14
Start Advance Unknown Additive

Advanced

Sefo's dad gave him $38. Now he has $94.

How much did he have before dad's gift?

Sefo has made a mistake.



Explain to Sefo what his mistake was.

2. Show how he should have worked it out.

ITEM 15

Advance Additive

Proficient

Is this correct?

262+90+38=390

Yes no

Which numbers did you add first?

Is this correct?

62+120+137=320

Yes no

Which number did you add first?

ITEM 16

Is this correct?

1406+1004+172=1471

Yes ◯ no ◯

Which numbers did you add first?

☐ ☐

Is this correct?

1506+2004+192=3691

Yes ◯ no ◯

Which numbers did you add first?

☐ ☐

ITEM 17
Advance Counter

Proficient

Is this correct?

6+4+7=18

Yes ◯ no ◯

Which numbers did you add first?

☐ ☐

Is this correct?

3+5+7=14

Yes ◯ no ◯

Which numbers did you add first?

☐ ☐

ITEM 18
Early Additive

Proficient

Is this correct?

17+9+23=50

Yes ◯ no ◯

Which numbers did you add first?

☐ ☐

Is this correct?

6+24+18=41

Yes ◯ no ◯

Which numbers did you add first?

☐ ☐

ITEM 19
Early Additive
Tens Pattern Grouping

Basic

Here is a number pattern

13 + 9 = 22
13 + 19 = 32
13 + 29 = 42
13 + 39 = 52

Write down the next line in the pattern

Use the pattern to work out what

13 + 79 =

Explain how the pattern works

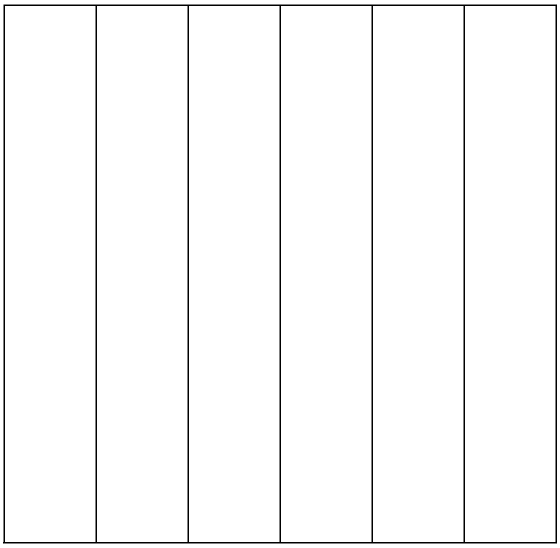
ITEM 20
Fractions Advance Additive

Rm 5's Swimming Sports

For room five's swimming sports they divided the pool into 6 equal lanes. There are 30 children in room 5. There needs to be the same amount of children in every lane.

How many children would be in each lane? □

How many children are in five-sixths of the pool? □



Swimming pool with lanes

ITEM 21

The Williams Family are having a pizza party. The “Pizza Palace” recommends the following servings:

Pizza Palace
Recommended Servings
(per person)

Adult	4 slices
Child 11+	3 slices
Child <11	2 slices

Pizzas have 8 slices

Show how you would work out how many pizzas you need to feed 2 adults,

4 children 11 and over, and 6 children aged 10 and under.

ITEM 22

Work out this problem

$$30 + 56 = \boxed{}$$

Which is the first step that came into your mind to work out this problem?

3 plus 5 is 8	56, 66, 76, 86
0+6=6	56, 57, 58……
Another way	

ITEM 23
(To distinguish advanced counters/early part-whole)

Sam and Loa were asked to solve 8 + 6.

I did 9, 10, 11, 12, 13, 14.

LOA

I did 8+2=10
10+4=14

SAM

Choose either Sam's or Loa's way to show how you solved this problem quickly.

1) 9+7

I did

The answer is

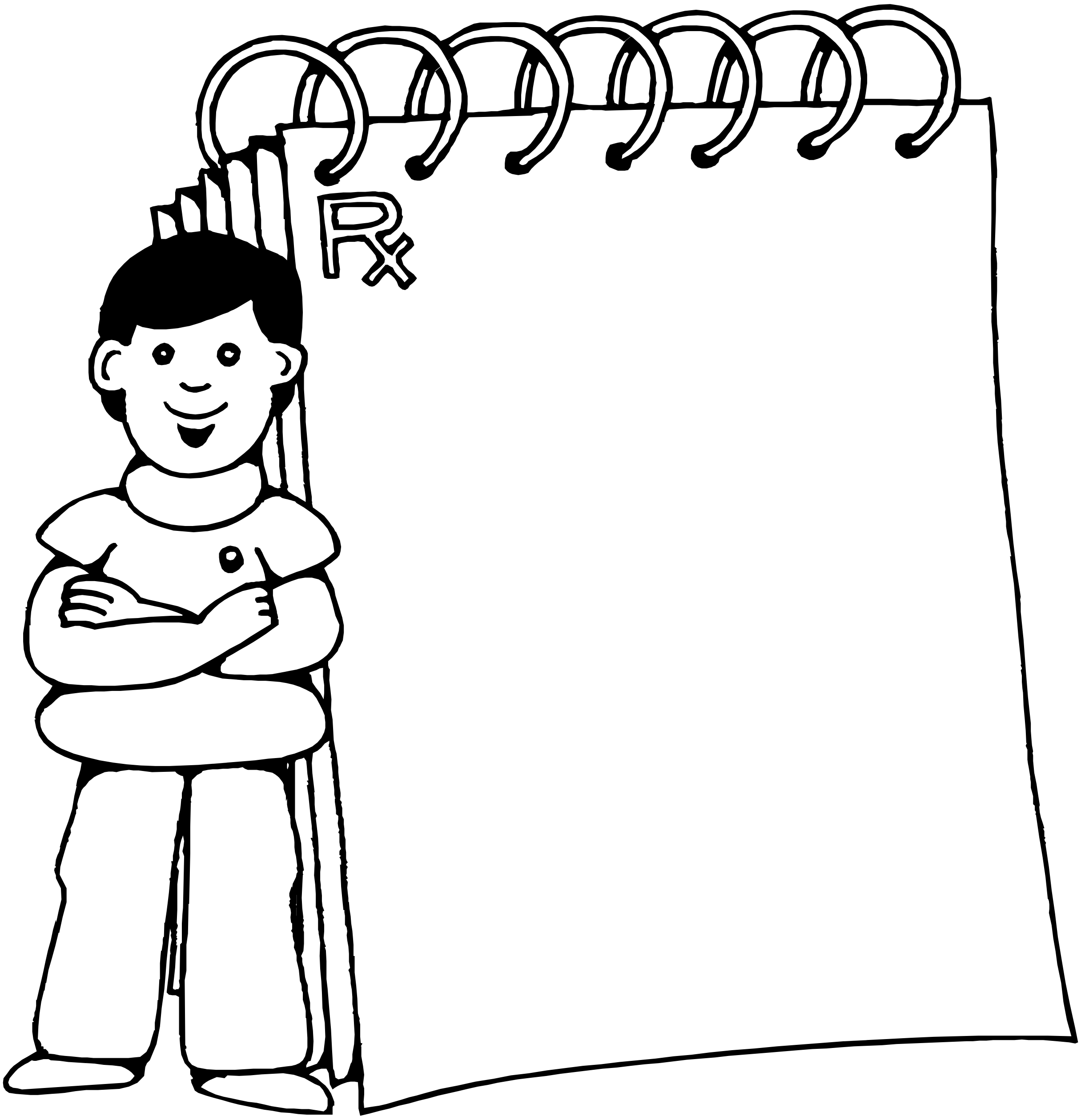
2) 38+8

I did

The answer is

ITEM 24
(Advanced Multiplicative)

Nadia thought ¾ of 28 apples was 7. Was she right, why or why not?



ITEM 25

To distinguish between skip counting-advanced counting or early additive or advanced additive.

I don't know 4x5. Which clue is the most helpful?

5, 10, 15	so	
5+5+5=15	so	
5+5=10	so	
3x5=15	so	
2x5=10	so	
4x10=40	so	
5x5=25	so	

ITEM 26

N L4-7

49.9 x 58.613

What is the best estimate for this problem? Do not actually calculate the answer.

a lot less than 3000

a little less than 3000

a little more than 3000

a lot more than 3000

Give your reasons for your answer.

ITEM 27

Early Multiplicative

Tom tried to challenge Martin with a hard problem to work out in his head.

99 x 13

Within 3 seconds, Martin gave the correct answer – 1287!

Tom was shocked he was so fast.

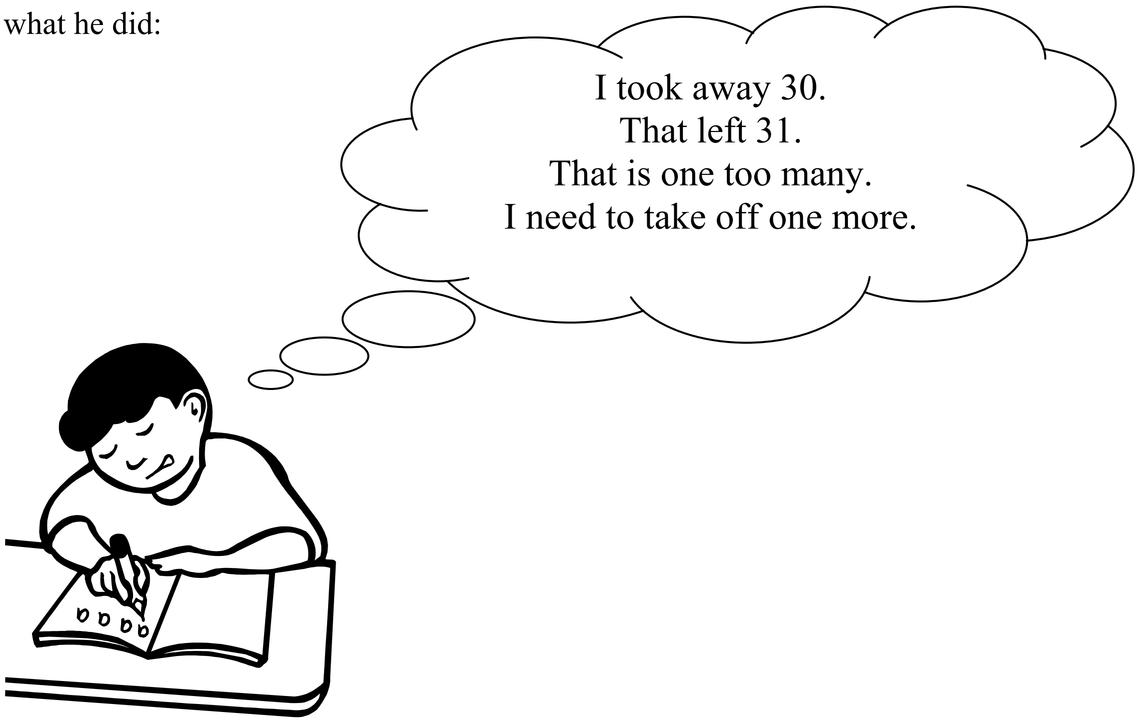
How did Martin do it so quickly?

ITEM 28

Peter was solving this problem:

61-29

This is what he did:



Will he get the correct answer?

Explain why.

ITEM 29
Place value

Advanced Additive

Advanced Level – Year 5 & 6

There were 894 seats at the movies. Only 548 were filled. How many seats were empty?

Show how you solved this problem.

Can you do this in another way?

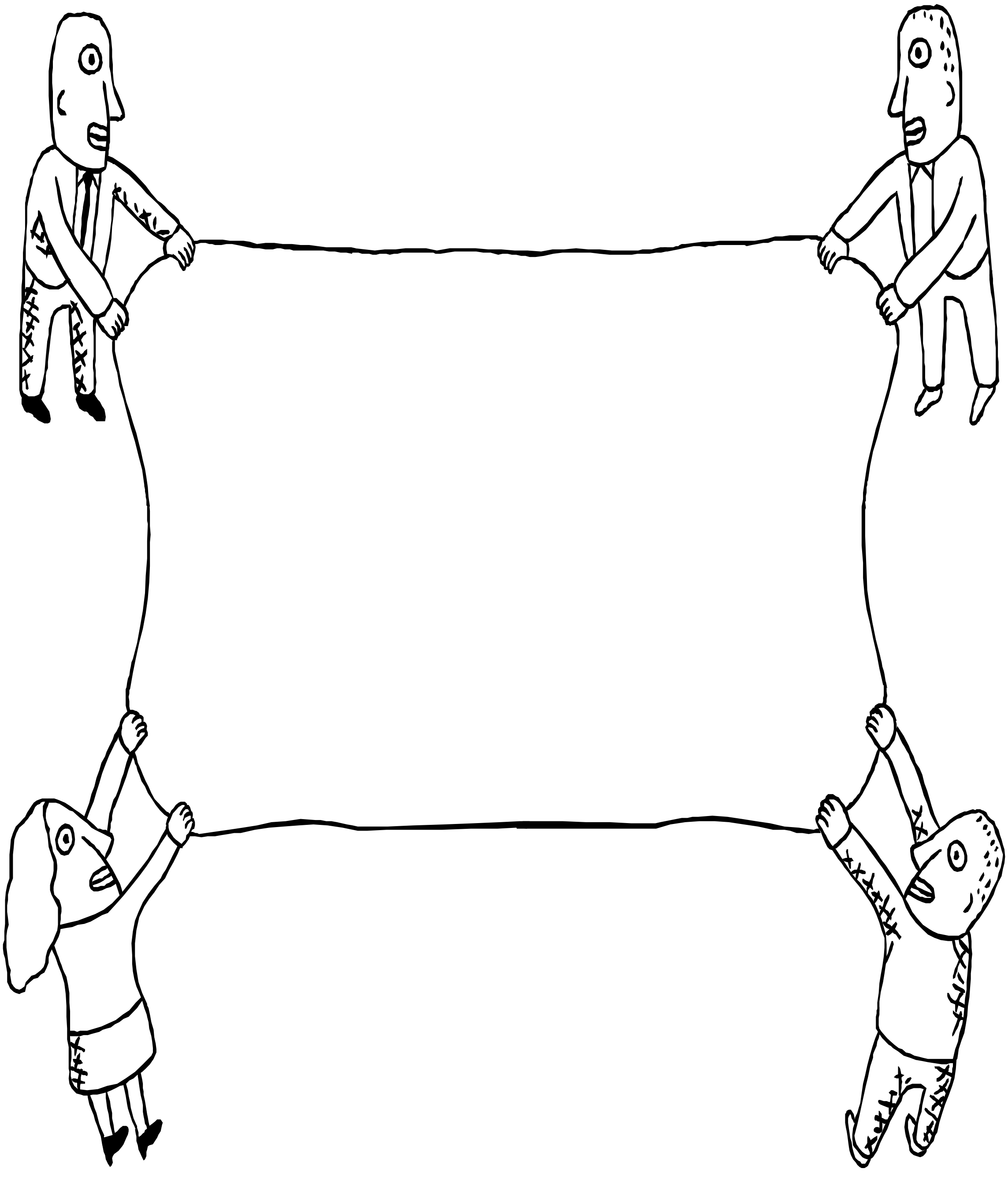
ITEM 30
Early Additive (place value)

Proficient Level

Year 5 & 6

Jo said that 36 and 48 is 84. Is this true? Yes/No

Explain how you would prove this.



ITEM 31
Advanced Level

Solving Subtraction at Advanced Additive

Year 5 & 6

There were 812 trees in the forest. 379 blew down in a storm. How many are still standing?

Mary said:

812-379
=812-380+1
=481

Is she right? Yes No

How would you do it?

ITEM 32
Distinguishing between Counters / Groupers

Year 5 & 6

Basic

8 + 8 is the same as

6 + 9

7 + 9

8 + 9

7 + 7 is the same as

8 + 6

8 + 7

8 + 9

16 + 16 is the same as

15 + 14

15 + 16

15 + 17

9 + 7 is the same as

9 + 9

8 + 8

7 + 7

ITEM 33

Distinguishing between Counters / Groupers

4 + 6 is the same as

5 + 5

4 + 4

6 + 6

13 + 15 is the same as

13 + 13

15 + 15

14 + 14

17 is the same as

9 + 8

9 + 7

9 + 6

28 is the same as

19 + 8

19 + 9

19 + 7

146 is the same as

139 + 6

139 + 7

139 + 8

ITEM 34

Your teacher has asked you to mark some work from another class. The first response to the problem looks like this:

$$67 + 34 = 101$$

Is this correct?___________________________

Show how you checked.

What is another way of checking the answer?

ITEM 35

Look at this question:

A school has five classrooms. Each classroom has four windows.

How many windows are there altogether?

Jill wrote this number story for this question.

5 + 4 = 9 windows

Is this the right number story for this question? YES NO

If you don't think Jill is correct, think of and write your own number story for this question.

ITEM 36

Jim and Lee each had some bananas.

Jim had three more bananas than Lee.

What would happen if Lee got another two bananas?

Who would have more bananas then?

Explain your answer in words or pictures or numbers.

ITEM 37

Five children share some apples.

They get three each and there are two left over.

Show how the apples are shared.

How many apples are there altogether?

ITEM 38

$3 + 5 + 7 = 14$

Is this correct?

Write down how you checked.

Write down another way of checking the answer.

ITEM 39

The students in a Year 3 class were told to work out 27 + 23.

Kiah did it on paper and her answer was 410.

Is Kiah right?

Explain what you think she did wrong.

Appendix B: Student Responses

Item 1

Year 7

C Read through the question two times. The question is easily done. If Paul had more money. If Paul had 4 times more dollars than Elena, it would work out.

P Same answer as C. Thought of an example it could be. How much money each could have. I thought Paul had $100 plus cause heaps more than Elena.

T This can be easily done. If ¼ Paul's money than ½ dollars of Elena. If Elena had $10, total spent ½ would be $5. Paul had $60, total spent ¼ which would be $16.

Year 6

H Yes, because Paul could have $100 and Elena $20 – ¼ Paul's is $25 but ½ Elena's is $10.

E Same as H. Just said, because it doesn't say how much money they have he could have one million and she 5 cents.

R Same as H. If like Paul had higher number she would have lower number. Doesn't matter if Paul had more.

Year 5

N Paul has more money than Elena. Problem says she spend ½ of the money and he spends ¼ money. Paul spent more must mean he has more.

J I didn't get it. If he spent ½ and her ¼ she still spent more. Didn't really understand the question.

K Paul had most money to start with. Figured out because Elena ½ and Paul ¼ and Paul spent more he must had had more money.

Item 2

Year 7

C Hard to get down on paper and explain. Worked out average of 2996 and 2998, which is 2997. Then plus 3000 will be 3 off. Then times 2997 needs to be plus 3 numbers to get to 3000.

P Thought 2996 + 2998 is same as 2997 x 2 because if you give one of the 2998's to 2996 they are both 2997. 3000 is three more than 2997 and it's like doing 97 x 3 plus another 3. Really confusing but I understood the question.

T Couldn't see how Sally worked it out. Had Gavin's answer. Understood the question but don't know how Sally could have known.

Year 6

H Because 2996 in Sione's is less than 2997. 2998 is higher than 2997. If count 2997 to 3000 it's 3. I ignored 2996.

E He has 2997 x 3 and she has 2996 plus another 2998 plus 3000. [Added numbers together to see how Sally knew that Sione's number was bigger than Gavin's.]

R Didn't go 3 x 2997 because next one is "plussing" it. 2997 + 2997 same as 2996 + 2998 equaled the same thing. Sione's is bigger because plus 3000 is 3 bigger because of 3000.

Year 5

N [Couldn't write explanation down. Asked to explain answer orally.] Because, 2998 take one of that and put is one 2996 becomes 2997 and Gavin's problem is 3 x 3997. Already done 2 but here says 3000 and the difference is 3.

J I did 3 x 2, 3 x 9 twice and then added them up and then 3 x 7. That's only a three digit number and Sione's is a four digit number.

K Gavin's must have been 2994 + 2997 + 3000 in another way of saying it.

Item 3

Year 7

C Did the last one [choice]. Looked at 399 and 601. Plus 1 to 399 makes round number. 400 + 6000 = 1000.

P First thing I wrote down – numbers one above another [399 + 601].

T Same thing as P's – write numbers one above another [601 + 399].

Year 6

H Put number one on top of each other. Then checked by putting 1 on 400 and then added together.

E Wrote down number and "plussed" in my head. 1 + 9 = 10 [and so on vertical form.]

R Wrote numbers and added them. Put them into columns.

Year 5

N Took one off 600 and put on 399 which makes 400. So 400 + 600 is 1000 – just easier to do it this way.

J Just put 300 and 600 = 900 and 99 + 1 is 100. [Realised answer wrong in box so change it.] And then 100 + 900 is 1000.

K Wrote numbers one above another. [Vertical form written out.]

Item 4

Year 7

C I chose Peter's way. 79 and 6. 6-1 =5, round number. 79 + 1 is 80, rounding. 80 + 5 is 85 which is the answer.

P Peter's way. Worked out exactly like his way. [Worked out answer before saw choices.]

T Saw pile of 79 sweets and 6 sweets would have counted in the head – 79, 80, 81, 82, …. [Saw T using fingers to count-on.]

Year 6

H One off 6 put in 79 made into 80 and then plus the 5. I did it Peter's way.

E Did it Jemima's way. 79 + 6 add the one equals 80 plus 5 = 85. [This way in fact was Peter's way – after asking again E stated he used this method to check his answer. He did use Jemima's way initially.]

R Just counted in my head 79, 80, 81 … . But to check if right I did it in columns.

Year 5

N Peter's way. Because take one off 6 and add it to 79. That makes 80 and 5 left over so it makes 85.

J Peter's way. 79 + 6 is 85. That's the way I worked it out. [Changed it to Jemima's way which is the way he explained it.]

K Jemima's way because it is the easiest way. Wrote 79 above 6 and add. Add 6 and 9 = 15, add 7 and 1 which is 8.

Item 5

Year 7

C Before I looked at the choices, did 51 + 49 in my head = 100. Knew it was 50 + 50.

P Turned it into 50 + 50.

T 1 + 1 = 10. Wrote it and worked it out.

Year 6

H I did it like the same one before. One off 51 to make it 50. So 50 plus 50 is 100. I like doing it this way best.

E Turned it into 50 + 50. Just went 49 + 1 = 50, + 50 = 100.

R 4 and 5 is 9 but actually I did 40 + 50 is 90 and then 9 + 1 is 10 and add it to 90 which is 100.

Year 5

N Took one off 51 put into 49 made 50. So 50 + 50 is 100.

J I did is 1 + 9 is 10 and then made it 50 from 49 and 50 and 50 is 100.

K Add 9 and 1. Then added 50 and 40 plus 10. [Explanation different from what written.]

Item 6

Year 7

C Thought a lot at the start and crossed heaps out [of his attempted answers]. 24.8 is almost 25. 25% means out of 100 or 1000. So 25 needs to go 4 times into 100. So actual number of rotten apples needs to be ¼ of 2000 so it's around 500.

P I did 81 times 24.8 which equaled a lot more than 2000. So knew that's probably why [Laurie knew answer was correct].

T 2000 is one whole. 24.8% so 1 in whole would be 2. 24.8 x 2 – that would be too small. [Re-worked the question at this point.] Changed line of thought. Not entirely sure how to convert percent into whole number. … 248 being answer – don't know how knew. Converted % into whole number. … In converting decimals into percent keep some but put percent or decimal point and change around. I thought 24% could be 248 whole number.

Year 6

H If 81 were rotten the percent would be higher than 24.8%. Because in SRA when we mark our work we mark in % and if we got 81 right we would get a % of about 63% so that's what I think. It would be more than 24.8%.

E [Does the line, dot, dot mean percent?] Percent goes into 100 and there's 2000 apples that means 20 100's so can't be 81 apples because 20 x 24.8% doesn't equal 81. It would be more apples.

R I did 81 + 24.8 which isn't 2000 so can't be that you take 24.8% away from 2000 because 81 wouldn't left so wouldn't work. (Haven't done percent in class yet so the question is sort of hard.)

Year 5

N Said 24.8 % of 2000 couldn't be 81 apples had to be more because 50% of 2000 is 1000. Like J said. I went 25% of 2000 is 500. Says 24.8 so can't be 81.

J Didn't get it at first. Then 50% is 1000. So 25% must be 500 so 24.8% must be wrong because 81 is no where near 500.

K It couldn't be right because only 24.8% out of 2000. It couldn't be 81 as it is too many apples. If more apples were rotten it could be right. It didn't say enough isn't 81 because 24.8% isn't 81. 24.85 – not sure [how many it could be].

Item 7

Year 7

C Looked at 8 and 4 and how could go into 80 and 15. Found separated 15 into 10 and 5. Found 5 went into 80 four times and 10 went into 80 eight times. Then added together and you have 12 which is the answer.

P Don't know. Couldn't work it out. [Asked to think aloud and what problematic – couldn't articulate as found question confusing.]

T Maybe calculation were 80 divided by 10 would be 8 and half 8 would be 4.

Year 6

H Think that 15 into 100 is around 1/3 of that so ½ of 80 or something like that.

E 10% of \$80 is \$8. 5% of \$80 equals \$4. Then 8 and 4 equals 12 dollars.

R Don't really know. To work out 15 – 80 see how much percent is left and its 65. Seeing, because says 15 into 15% of 80 is 12, 15 x 12.

Year 5

N Saw one answer 8 and other 4. First did 8. Must be ½ and second calculator must be 4. [N got stuck at this point. Indicated he understood the question but couldn't work it out himself.] I could have worked out 16% because 8 ½ of 16 and \$4 ¼ of 16.

J Don't understand the question. Well, one of them is 8. Calculation must have been a high number from the start. She added 4 to eight but I don't known how she knew how she added the 4 and 8 to make 12.

K If she wanted to work out 15% of 80 could have done 80 divided by 15 and would have had the answer. If had two different answer could have done 8 divided by 12 and would have been 16. I don't understand where got the two calculations from.

Item 8

Year 7

C Took 3 from 13 then 10. Took one more away to make 9.

P Went 9 + 4 is 13.

T I thought basically … used a different method than on the page. Asked 4 plus what is 13.

Year 6

H I took, since this is a small number I automatically knew it was 9. 3 from 13 leaving 10 as it was the closet one to my answer.

E Just minus 3 which equals 10. Minus 1 equals 9.

R [Was using fingers to count.] I went 12, 11, 10, 9. Just went 4 down and the answer is 9.

Year 5

N Took one off 4 and made into 3. Then "minused". It because 10. Then 1-10 is 9.

J [At first added the numbers together resulting in him writing another strategy down. Starts explaining his strategy when he notes his error. He did not work out the problem but took his answer from the first strategy noted. His written answer, subsequently, makes no sense.]

K At first I tried to stick the numbers on top of each other [vertical form] but knew that wasn't the right answer. [She had difficulty when trading and changed the three to four rather than three to 13.] So I went 13, 12, 11, 10, 9 and knew 9 was right.

Item 9

Year 7

C Worked out first [before saw choices]. It must be one more than 3 cause 13 and 9 so 1 more than 3 has to be 4. One more to be 10 + 3 is 4. Made into 10 and saw 12 so needed 3 more.

P Remembered that from last question because basically they are the same. Thought 13 – 4.

T Just thought 9 + 4 is 13. [Wrote down equation.]

Year 6

H Would have done it using c [3rd option] but the last page was the same so 9 + 4 = 13.

E 9 + 4 = 13. I just knew it.

R Just went the same way as last time: 10, 11, 12, 13. To check I went 9 + 4 = 13.

Year 5

A Just added 1 onto it [9], 10 + 3 = 13 and added 1 onto 3 which made 4.

L I added 1 to 9 then added on 3 more that made 13. Put 3 and 1 together which made 4.

S Had 9 then, easy way to do it is plus 1 and add on 3 to make 13 on 1 to plus makes 10.

Item 10

Year 7

C Did choice 3 – exactly that way. Just knew 29 plus 5 is 34 caused of number three choice.

P 9 + 5 is 14 and add on 20 is 34.

T First choice [counting on].

Year 6

H 29 + 5 = 34. If plus 6 on 29 would be 35. But I used the method I told you before. You know how I did 79 + 6 = 85. "Plussed" 29 with 5 but I knew it off by heart.

E I actually knew what the answer was already. Just remembered it from another time [during maths].

R Rounded 29 off to 30 then "plussed" 5 which is 35 and then took away 1 to make it 34.

Year 5

N Minus 7 from 8 add to 29 which is 30. That leaves four left over to add it to which is 34.

J Minus 1 from 5 add to 29 which is 30 + 4 is 34.

K Took 1 off 5 and that's 30 and 4 more and 30 and 4 is 34.

Item 11

Year 7

C Exactly same as P – word for word. Just copied the top [method] just used different numbers.

P Did $1.95 is close to $2. $2 times 8 = $16. $1.95 is 5 cents less than $2. 5 times 8 equals 40. $16 - .40 equals $15.60.

T Well, $1.95 is very close to $2 times 8 equals 16. So 5 cents left out. 5 times 8 = 40. So $16 - .40 is $15.60.

Year 6

H Make it cost $2 each then minus 40 cents. Instead minus 30 cents … $6 is two less than 8, 6 is 30 and 8 is 40. Because of that know 2 is in between. Then 1 off 2 will make five and other make 10. So 30 plus 10 is 40. [I read back this to H word for word, she said I had written it down correctly!]

E Well, 2 x 8 = 16 – 40 cents = $15.60. [To get 40 cents] added 5c to make it $2. Then 80 divided by 5 = 40 cents. So then equals $15.60.

R Went 1 x 8, cause like 1 more than 8, so $8. Then added .95 and .95. So five and five is 10 and 9 and 9 is 18. I changed to 180 because working with 90's not 9 and then 90 x 4. Well that way I had 1.90 just plus 1.90, so 8.

Year 5

N $1.95 is close to $2. So $2 x 8 = 16 but put extra 5 each time so 5 x 8 is 40. So "minused" 40 from $16. So equaled $15.60.

J $1.95 is close to $2 so 8 x 2 is 16. 8 x 9 is 73, 8 x 5 is 40. So 16 + 7 which is 23$ and 40 cents and 3 cents is 43 cents. I think the answer is $23.40.

K She must have worked it out the same way as before because she got a quick answer. [Could you follow how Grandma did it?] No. [Did observed K trying to follow method but she couldn't do it. She found it difficult because of the answer she got - $8.15 and it couldn't be $81.05.]

Item 12

Year 7

C 20 divided by 4 equals 5. [How arrive at that equation?] 20 mice, 4 holes therefore 20 mice divided by 4 holes equals 5 mice.

P Almost same thing but wrote 4 times what = 20. So 4 times 5 = 20. So five in each hole.

T Did same as C – word for word.

Year 6

H 20 divided by 4 = 5 because 4 x 5 = 20.

E 20 divided by 4 = 5. Because 20 mice, 4 mouse houses and then I just divided 20 by 4 = 5.

R Well I did how many times does [repeats question.] 20 divided by 43. Then tried 5. Went 5, 10, 15, 20 – that made four times that made it five in each hole.

Year 5

N There was 20 mice and 4 mouse holes so when 20 divided by 4 = 5 because it said the same numbers went into each hole.

J Counted up in 2 and ¼ of 20 is 5. So five go into each mousehole.

K Said 4 x what = 20. Something times 4 = 20. Realised 5 x 4 = 20. Realised this ties into times tables, so just times.

Item 13

Year 7

C Saw rounded numbers into next tens and then up the number into the tens like 50 to 60 and then added ones again. Rounded to tens and then added ones for all questions. Again just copy top [method] but with different numbers.

P Made big line to 60 then 66. 66 + 4 = 70 and then 70 plus 10 is 80 + 2 is 82. Just did the same thing for all of them [questions].

T 66 to 17 to 82. 197 to 7 to 204. 1789 to 13 to 1812. Worked out problems down the side. [Observed T counting on fingers.]

Year 6

H [Asked if had to use number line. Instead used vertical form.] First 82 – 66 = 16. Since got answer did the number line. 0 to 66 then try and figure out before can add number like 10 and 5 and whatever until equals 82, but I knew the answer. [Second question.] I put 197 up to there then 3 then 4 that was 201. Different than the top one. If 3 from 4 200, one left over equals 201. [Third question.] Last one I found it a bit hard. Put 1798 up to there. Round 1781 to 1790 then to 1811. Added 11 on to it then 10. Just figured out and tried best from there.

E Just did same as on number line [example]. You go from 66 to 70 then put 4 in. Then 70 to 80 that equals 10. Then 80 to 82 then put 2 in. Add all together equals 16. Same method as for rest of them.

R [Stated couldn't use number line at all. Did not attempt to use them at all rather used vertical form to solve equation.] Just put the 82 – 66. Just worked out in columns and equals 16. Did all of them like that. [What's confusing about the number line?] Don't see how can go from 0 to 48. Don't see the pattern between them because 50 + 6, 10 between there, 2 there, but 2 and 2 doesn't make 10. Only makes 4.

Year 5

N 66 added 4 made 70 and then from 70 to 82 distance of 12. Added that which made 82. [Second number line.] 197 + 3 made 200, added 4, made 204. Added the numbers 3 and 4. [Third number line.] Got confused. I added 1 which made it 790. Then added 22 to it. Made 1812. Added 22 + 1.

J Remembered added 20 would be 86. Found out was 5. 60 + 20 is 80. Then knew 6 on end couldn't be the answer. I did it again. Number less than 28 and 16 on. 66 + 14, forgot 2 over here. [Basically J was quite confused at this point with the number line.] [Second number line.] Added 5 to 7 made it 200 and added 4 to it 204 and 3 and 4 is 7. Number line 197 + 7 is 204. [Third number line.] Did 1789 plus 13 goes to 1812. First thought that both number were same – looked at hundreds. 700 changed to 800 but added up and it was right. 10 less from 12 from answer, so added another 10.

J I don't get it. [Took while to figure out how to use number line but eventually did.] 66 plus 10 is 76 plus what is 82. So 76 plus 6 is 82. [Second number line.] 7 and 7 is 14 so must be 10 added onto 200 so 204. [Third number line.] Don't get it. Can't work it out. Tried to see how the answer was changed into 800s. But it is a big number and I can't add it up that easily.

Item 14

Year 7

C Did the same as P – 94-38 = 64. Found 32 was used a lot and maybe 32 + 32 = 64 but I don't know what he was doing. [Realised he was doing something wrong during his explanation. With P guiding him, he worked out what to do.] Thought it was 8-4 rather than 4-8.

P I did 94-38 = 56. [Sefo's mistake] – he added 94 and 38 instead of taking away. He must have less than $94 before and given some. Now he has $94 so can't have more.

T Thought mistake 38 + 94 – he has $94 now and dad gave $38 – how could he have $94 before gift if he has $94 now. Should have been 56 + 38 = 94. [How did you get $56?] 94-38 = 56.

Year 6

H He wasn't meant to add on 38 to 94. Dad gave only 56 dollars so if add on he only had 94. Should have worked it out 94 – 38 dollars.

E Just wrote down 94-38 = 56. Sefo's mistake, all up, included money from dad. He had 94 so he didn't need to add anything to it.

R His mistake, he "plussed" 38 and 94 instead of "minusing". I worked it out in columns: 94-38.

Year 5

N Mistake was 38 + 94. Should have done 38 – 94. Would have had the answer. I did $56 already to 38 added 2 and made it 40 then I added 50 to make it 90 and only 4 left over. Added together made 56.

J I actually added the answer wrong. Add 30 + 90 which is 120 and 12 is 132. I don't know what's wrong about Sefo's problem. I think it was right.

K [Originally going to leave question as it didn't work when she put the numbers into vertical form.] I put the numbers on top of one another the first time and did a mistake now it's right. 94 - 38 = 56.

Item 15

Year 7

C Added 90 and 38. I don't know why it just seemed right. Think it was to make 100. Added 128 and then added 262 and was right. [Second problem] – Continued same pattern: 120 + 137 then plus 62. Picked 120 + 137 as followed same pattern as first question.

P Just added them all up like that [vertical form]. So first question 2 and 8. Second one, 2 and 7.

T Thought the first one was right. I added 262 + 38. Thought the second one was wrong. It didn't look right. Added 120 + 137. [Why did you pick those two numbers?] Second number might give key to answering them as looked more relevant.

Year 6

R I added 60 from 262 to 90 and then just added on the 38. [Second problem.] Wrote it up like proper additional form and started with the 2 and 7.

M Just did in highest order to lowest [larger to smaller number]. Put into columns and added 2 and 8. [Second problem.] Did exactly the same, highest to lowest, and the two numbers were 2 and 7. Did it from highest to lowest in that order as it is easiest.

A First added first two numbers 262 and 90. Then added answer to 38. Chose number first and wrote them down in columns. [Second problem.] Took away 2 from 60 then added 60 to 120 and then added 2 to that answer then + 137.

Year 5

N First one went 262 + 90. If change 90 into 100 will make 362 take away a ten, 352. Second one, did 120 + 62 = 182 plus 7, 289, added on 3. 120 and 62: ignored the 2, 120 + 60 equals 180 then added 2.

J First one, 262 + 90 then added up left in head then added 38 to that = 390. Added 262 + 90. Took off 40 from 90 put on 60 made another hundred. 50 left from 90 made 352. Second one, 60 + 20 was 80, added 2 to that, 182. Added 137 to that = 317.

K First, wrote down the numbers on top of each other. Added 2 and 8 which is 10 [continued explaining how added numbers using vertical form]. Second one, Didn't get the same answer. I added 2 and 7 [continued explained how added numbers using vertical form].

Item 16

Year 7

C Same as before. 1406 + 1004. Chose those as they were the larger numbers and seemed to go together as 6 and 4 is 10. [Second problem.] 2004 + 1506 – same reason as I already said.

P Did the same way: added 6 + 4 first and 6 + 4 on the other one because 6 and 4 is 10 and easy to add and as they are the first numbers due to the way I wrote out the answer.

T First one correct. Added 1406 and 1004 – both are similar numbers. [Second problem.] Thought correct. Added 1506 + 2004 because seemed sequential.

Year 6

R I added 1000 and 1000 which is far too high for the answer. Next one added 1000 to 2000 to make sure not too high. Did oral check which was adding estimates of numbers – too high so did proper check in columns. Same, too high.

M First saw couldn't be the answer because two 1000's in question but the answer is less than 2000. Then added. [Second problem.] Didn't know just worked it out [vertical form].

A Just first added 1400 to 1000. Then added 6 + 4 = 10 and added 10 to that. Then wrote down answer. Saw couldn't equal 1471. [Second problem.] Added 1500 to 2000. Then added 6 and 4 to that then wrote down in columns - to that answer added 192.

Year 5

M Just knew 1406 + 1004 was bigger than 1472. Added together by putting on top of one another. [Second problem.] First 1506 and 2004 [added using vertical form]. Then it was pretty close [the answer] so had to do the third number.

D First knew that is wasn't because 1406 + 1004 equaled too much – wouldn't be 1472. Added on top of each other [vertical form]. [Second problem.] First 1506 + 2004 because "plussed" it equaled 3510 then "plussed" with 192 = 3702. That was too big. Numbers added, 1506 and 2004. Chose them because they were the first ones [in the equation].

G Knew too much "plussing" together. To make sure, did it downwards, it was 2582. First numbers added, 6 and 4 because going down [vertical form]. [Second problem.] Wrong as well because if actually "plussed" all together wouldn't equal the amount. Added 6 and 4 because downwards [vertical form].

Item 17

Year 7

C Added 6 and 4 because they make 10. Then 7 and 17. Wasn't right [the question]. [Second problem.] Added 3 and 7 because make 10. Wasn't right [the problem].

P Added 6 and 4 because added u to 10 then added 7 and 17. [Second problem.] 7 and 3 because added up to 10.

T First one incorrect. Added 4 and 7 because thought they could become a number that could be added 6 more on more easily. [Second problem.] Added 3 and 5 because when put together could be a number closer to 7 and could add 7 more on a lot easier.

Year 6

R Beginning bit I chose 7 and 4 could use 6 for easier addition. Second one, chose 3 and 5 and saw odd and even don't equal even number so saw wrong. Chose 3 and 5 because at beginning of problem.

M Chose 6 and 4 because know if equal 10 than another number I always know the answer. [Second problem.] 7 and 3. Chose them because = 10. Had 5 left so knew it was wrong. 7 and 3 because add to 10.

A Just in my head knew this [6 + 4] = 10 and 7 did not equal 18. Chose those because first [in equation]. [Second problem.] Just added 3 and 5 = 8. Looked at 7, couldn't be 14. 3 and 5 were chosen because at beginning [of equation].

Year 5

M First one, it was wrong. Added 7 and 6 first because the are the highest numbers so don't have to work out higher number. Knew 12 and just have to add on 4. Second one wrong. Did 5 + 7 because of one, 7 + 6. So added another one and then just added on three.

D First one wrong because 10 + 7 = 17. Added first 6 and 4. Chose 6 and 4 because knew it equaled 10 and knew 10 + 7. It was easier for me. 5 + 3 = 8 and 7 + 7 = 14 not 7 and 8. Numbers added first, 3 and 5. 5 + 3 is 8 and 8 + 7 is not 14.

G Went 6 + 4 + 7 is 117. That's not right. First numbers did 6 and 4 because equal to 10 and it's easier. [Observed G using fingers and counting on.] Second one, did 3 + 5 is 8 + 7 equals, it is too much because it's 15. First ones, 3 and 5 because first one [in equation].

Item 18

Year 7

C 17 + 23 (wasn't correct). Because 7 and 3 equals 10 which is easier to deal with. Chose number because of one's column. Chose 6 and 24 because equal 10 again – one's column.

P First one: added 3 and 7 which is 10. Second one: 6 and 24. Did this because seemed real easy because 6 and 24 is 30.

T First one: incorrect. Added 17 and 9 together become a number close to 23. Second one: incorrect. Added 6 and 24 because I thought they would become a number 8 could easily be added in to.

Year 6

R First question, 9 and 23. I am quite good with 9's. Added 17 and saw it was wrong. [Second problem.] 6 and 24 because 4 and 6 make 10. Be easier, so I knew it [problem] was wrong.

M 23 and 17. 7 and 3 = 10 and another 10 would be 40 plus extra 9. [Second problem.] 24 and 6 would be 30 so just another 18.

A Took away 7 from 17 and took away 4 from 9. Added 10 + 5 and 7 + 4 and got my answer and then 23 to my answer. Just basically knew 24 + 6 would = 35 and then I took away 5 from 35 and took away 8 from 18. Added 30 to 10 and added 4 + 8 and added 40 to the answer 8 + 4.

Year 5

M First one "plussed" 23 + 17 because higher numbers again and got 40 and knew 40 + 9 did not equal 50. Second one, 24 + 18 = 42 and then plus 6 more. Already gone over 41. [Did use vertical form to add the first two numbers chosen for each problem.]

D Did 17 + 23 = 40 + 9 + 49. That's too small. First numbers, 17 and 23 because they were highest ones added together [used vertical form to add]. Second one, did 24 + 6 = 30. 30 + 18, knew equaled 48. Chose 24 + 6 = 30 because 4 + 6 = 10.

G First one did 23 and 17 because knew it equaled 40 because I have done that before. Knew equal 49 not 50. Second one wrong because 18 + 6 is 24 and 24 and 24 is 48. Chose 18 and 6 because thought be easier and quicker to work out because 6 is a smaller number and 18 is on the 6 times table.

Item 19

Year 7

C I did 13 + 49 = 62. To use pattern to work out number is 92. I continued the patter to work out [problem] and checked it by going 13 + 79 = 92.

P Next line is 13 + 49 is 62 and then 13 + 79 is 92. Starts with 13 + 9 and goes up in tens. [To solve.] Just continued pattern but you could solve it a different way: 7 + 2 is 9. so 92.

T Next line figured 13 + 49 = 62 and for next one 13 + 79 is 92. Pattern works by adding/increasing numbers to one another. Also observed the first number is always 13 so 13 plus whatever 9 – that answer will always have a two on the end. [Asked how he solved 13 + 79. Stated he did not continue pattern to solve it just added the numbers together.]

Year 6

R First one, followed the pattern and added 10. Continued pattern to get answer to next one. End up with pattern, just add 10 to answer the 9.

M Used pattern to solve first question. Second question, 13 + 79 continued the pattern but by counting in tens. The pattern: go 10 higher than the answer.

A Some 13's going. Saw all had 9's on end. Worked out next one by seeing what number came next. Saw all had twos on end. 13 + 79, went, 7 and saw had 2 to it. Saw put down a 2 on the end. Plus 13 to 9 = 22 then add 13 to 19 (add 1 to 9) then 13 to 29 (add 2 to 9) and so on.

Year 5

M Looked at pattern. They were all 13. Knew next one 13. Second numbers 9, then 19, 29, 39 so knew next number 49. So plus 49. Looked at last numbers, so 62. Did 13 + 49 = 62, 13 + 59 = 72, 13 + 69, 13 + 79 = 92. 13 always in front then second number just added 1 and 2 and 2 and added from there. [Note, he stated he didn't understand the second part to the question and I observed M using vertical form to answer the question initially.]

D Used pattern to solve the question. All the numbers end with same number and it goes from smallest to largest on both sides.

G First 13 + 49 = 62. To solve 13 + 79, kept going down in a pattern [which she wrote out]. On this one, each … [explained numbers contained in columns for example 13 in first column and so on.]

Item 20

Year 7

C Divided it by 5. 30 children in room, 6 lanes and then divide 30 by 5 is 6. There's 6 lanes in the whole swimming pool so must be 5 – 30 which is 25.

P I did – 6 lanes, 30 in class so 30 divided by 6 is 5. Then just counted up lanes to 5. So 5, 10, 15, 20, 25.

T There would be 5 kids in each lane because five 6's are 30. So take away one lane: 5 x 5 is 25 because one lane doesn't count for that one.

Year 6

R In head divided 6 into 30. Ended up with 5. Next questions just put 5, 10, 15, 20, 25 in first five lanes.

M Put 6 into 30. Estimated. Got 5 so 5 x 6 = 30. So knew it right. 5/6 would be 30 take away 5 so 25. Estimated 5 because did quick 6 into 30. I knew the times table so I got 5.

A Did 5 divided 30 =, hang on, go mixed up always said 5. Because saw, 6 equals lanes instead of 5. Saw 6 instead of 5. Went 5, 10, 15, 20, 25, 30. Not 24 [as written on her paper] it would be 25. Take away 5 from 30.

Year 5

M Just knew 6 times table. So 5 x 6 = 30. So knew 5. Didn't understand it [second part to question.]

D First one divided these [dots she drew on page] into 30. So 5 in each. [Kept drawing dots for each lane until counted up to 30.] Second one went across and it came to 6. Really only guessed that number.

G For first one, knew 6 times tables so 6 x 5 = 30. Second one, just guessed so 6. Know 5/6 is 5 out of 6. So leave 5 lanes out of the six ones. So took a guess 6 children out of the 5 lanes.

Item 21

Year 7

Q Like, each person, 2 adults. Calculate how much per person. Used multiplication.

V Says 4 slices for adult, child 11+ 3, child < 11 3. I did this using an equation [showed working]. 4 x 2 adults = 8; 4 x 3 children 11+ = 12; 2 x 6 children < 11 = 12. Then added up to 32. 8 slices for 1 pizza divided by 8 = 4.

Z At beginning I didn't see that the pizzas have 8 slices, just did the number of slices. 4 adults x 2, 4 children 11+ x 3, 6 children <11 x 3. Added them up then divided by 8. So 4.

Year 6

R Put adults in so 4 x 4. Added all the kids, 2 12's. Added all up 32. So 32 divided by 8 = 4.

M Did adults, 4 slices 2 minus 1 pizza. Then 4 children. That would be 9. One left over, 2 pizzas. Then other children 6 slices. So 2 remainder one. So four pizzas. [Asked him to explain this process again as somewhat confusing.] Adults, 4 slices x 2 = 2 leftover. 4 children 11+ 3, 6, 9 – one. Another child 11+ 3 slices + 1 slice so 2 pizza and 1 pizza = 4 pizza.

A Worked out in my head. Just added 2 adults = 8. 4 children 11+ worked out to be 12. Added together. Then added 12 to that. Ended up with 32. So 32 divided by 8 = 4. So 4 pizzas.

Year 5

M I did adults, 2 x 4 are 8. So 8 and then did 4 x 3 for children 11+ = 12. Then children less than 11, 2 slices. So 2 x 10 supposed to 2 x 6 = 12 slices. So got 20 pizzas. 8 + 12, so 20 pizzas. No …. Yeah – I did something wrong but I can't find the error.

D First to feed 2 adults, 4 slices. So 4 + 4 = 8 whole pizzas. Children was 3 slices … hang on, didn't do it right, well did altogether – added all together. I didn't separate, got 19 slices of pizza for all children and 2 adults need altogether. So 2 pizzas and 3 slices.

G For the children less than 11, one and ½ pizzas because pizzas have 8 slices. 2 x 6 are 12 so 12 – 4 is 8. So knew 1½ pizzas. Children 11+, 3 slices. That would be 12 slices all together. Same thing, 1½ pizzas. Adults, 2 adults would have 4 pieces. 1 whole pizza. [Asked her how many pizzas altogether, just reiterated number of pizzas for each group which would probably be 4 pizzas altogether.]

Item 22

Year 7

Q I saw 0 + 6 is 6 then 3 + 5 is 8. [Saw this in her head in a vertical form.]

V I worked out that problem 30 + 56 is 86. Don't know how, did it in a chunk. [After some prompting she then acknowledged that she went 3 + 8 or 30 + 50 plus 6 at the end.]

Z Well says 30 + 56, take off zero and plus 3 + 5 and then add on rest.

Year 6

R Just added 3 into 5 and got 8. Put 0 on and added six.

M Same [as A]. 30 + 50 = 80 + 6.

A Just added 30 to 50 and then added six.

Year 5

M Did it in my mind: 3 + 5 = 8. Then plus 6, 86. Started with 3 and 5 because starting number. Imagined 5 was 50 = 80 + 6.

D 0 + 6 is 6 then 5 + 3 is 8. Found it easier [to do this method].

G Did on piece of paper and wrote it [vertical form: 30 + 56] down. 0 + 6 is 6 and 3 + 5 is 8 so 86. Found [this method] easier and much quicker to do it this way.

Item 23

Year 7

Q First thing, 9 + 7. Counted 9, 10 and 6 left over so 16. Usually count to 10 and put rest of numbers. This one [38 + 8], not sure how to do it so I did it like the girl did [counting on].

V Well, for number 1 to make, if two big numbers, 9 + 1 is 10 and 6 left over so 16 or literally 9 + 7 is 16. [Second question.] Did 8 and 8 is 16 and then 30 and 16 equals 46. One to tens this time.

Z 9 + 7, I got 9 is one number away from 10. 10 + 7 = 17 take away one is 16. Number two, 8 + 8 = 16 + 30 because 38 + 8.

Year 6

R First one, Loa's way. Because easy question. Second one Sam's way because it is a harder question and his way is better for harder questions. For 9 + 7, I would have automatically put answer. For 38 + 8 would have done 8 straight into 38. First column, second column, but also used Sam's method.

M Did Loa's way because I never learnt how to do it Sam's way. Would have worked it from head 9 + 7, oh, Loa's way for this one. 38 + 8 – Loa's way. But I would have written it down.

A Sam's way just worked out 16 and other one 46. Another way, for example 9 + 7. Take away 4 from 9 and 2 from 7. So 5 and 5 = 10 and 4 and 2 = 6. 10 + 6 = 16. 38 + 8 – take away 3 from 38, so 35. Take away 3 from 8 so 40 and added 3 and 3 = 6, so 46.

Year 5

M Found Sam's way very confusing. Would have use paper but only for 38 + 8. Used Loa's way to solve the problem in my head. Sam's way I understood but found confusing.

D Did it on my fingers. Would have done on 38 + 8, drawn it on the paper. Sam's way was pretty confusing.

G I did it with my fingers because found Sam's way confusing. Would have solve it by doing it on piece of paper [vertical form]. [Both D and G were confused by the question. Neither way really seemed to represent their preferred way for solving those problems.]

Item 24

Year 7

Q Well, I did working she had not completed as not times by numerator. I did it and came to 28 not 7. 28 divided by 4 would come to 7 but also has to times by 3. True answer is 21.

V She was wrong because ¼ would be 7 as 28 divided by 4 = 7. But there's a three. 7 x 3 = 21. So she did first half of question and didn't complete it. Then just got answer 21 because 28 divided by 4 = 7 and 3, which is a whole number. Multiplied it to 21.

Z Said wrong because ¼ of 28 is 7. ¾ of 28 is like going 7 is ¼ of 28 and times by 3. So time by the numerator = 21.

Year 6

R 28 divided by 7 = 4 and then she want ¾'s so already got 28 divided by 4 = 7 and she wanted ¾. Add on other 14. She ended up with 7. She didn't add on extra numbers. So she was wrong.

M Don't know. She was right. Tried to do 7 x 4. Don't know why. Didn't try anything else. [After M's explanation R taught M how to solve the problem, in fact he explained it far better the second time while teaching M than his first explanation.]

A Think she was wrong. Just guessing, oh no I think she was right because 3 x 7 = 28 so should be some sort of rotation. Did look at whole thing and that's ¼'s and ¾'s one. I've forgotten lots [about fractions]. 7 would be far too small because ¾ apple [pointed to picture] would be far more than 7.

Year 5

M Did 7 x 4 = 28. Told me it [the answer] was right. I did, I know that 7 apples and bottom number is 4 [3/4]. Tried 7 x 3 = 21 – wrong. So tried 7 x 4.

D Tried 7 divided by 28 but equals 4. Didn't try anything else. … Yes, because 7 x 4 = 28.

G Looked fun but not! It's hard. Fraction are like division, right? Tried 7 divided by 34. Couldn't do it. 34 came from ¾ because 3 out of 4. Just confusing.

Item 25

Year 7

Q First one, chose 5, 10, 15. If you keep skip counting with 5 and 3 lots of 5 is 15 and another lot would be 20. 4 lots of 5 is 20. Second clue, 3 x 5 = 15. Helpful because if you put 4 into 5 is 20 as 3 into 5 is 15. So it will be almost same as the top one.

V I have an argument against Z. Her way is the sleepy way but mine is not fast either. Chose the first clue 5, 10, 15. Not fast but can be accurate. She should know her times table. Second clue, 5 + 5 + 5 = 15. I chose it because if don't know times table should do 5 + 5 + 5 until you get the answer. This is 3 lots of 5, so 15. 5 x 3 is same as 5 + 5 + 5. Then 4 lots of 5 is 20. So 5 + 5 + 5 + 5 4 times will make 20. I think multiplication is about how many lots of something.

Z Number 1. 2 x 5 = 10. Because 2 x 2 = 4 so just go 2 x 5 is 10 and times it by 2 is 20. Second, 4 x 10 is 40 because the same thing only ½ is 5. So half of the answer is 20 the answer to 4 x 5.

Year 6

R Did 3 x 15 because closest to 4 x 5.

M Chose 3 x 15 because then can just do 4 x 5. Just adding another 5 if don't want to do times.

A Did 5 + 5 + 5 = 15 because three 5's = 15, 3 x 5 = 15. And then just another 5 and they will get the pattern. So add another 5 and get 4 x 5 = 20.

Year 5

M Chose 5, 10, 15 because easier counting in fives.

D Did 5 + 5 = 10 because all have to do is 5 + 5 twice and you would have the answer. Do 5 + 5 = 10 another 5 + 5 = 10 then plus it.

G Did 5 x 5 = 25 because 5 x 4 = 20. She would just have to take away 5 to get the answer.

Item 26

Year 7

Q I did it wrong as thought was plus so I had a lot less. So did 50 x 59 to get 2950. So 2950, so I chose a little less than 3000 as not that much.

V Think b [little less] because 49.9 is 50 and 58.613 is less than 5. So rounds to 59. 58 x 50 is 2950, is less than 3000. 60 x 50 is also ok if can round and then don't have to do any calculations but no exactly 3000 is answer so, did a bit less.

Z 49.9 rounds to nearest 50 and 58.613 rounds to nearest is 59 whole number. 50 x 59 is 2950 which is less than 3000.

Year 6

R Said less than 3000 because 49.9 closet to 50 and 58.613 closest to 6. So 50 x 60, take away 0 = 5 x 6 and that equals 30. Then put 0's back = 3000. Then take away amount added 1.497 amount and 0.1 added on. Took away from 3000.

M Little less than 3000 because 8 x 9 is 72 which is second one before the point. 5 x 4 = 20, first one. Then add 7 to the 20 and then still got all those [numbers] behind the point.

A Just looked at 4, oh no, though plus. Oh no doesn't matter still. Same because 49.9 plus 58 is less than 3000. Chose d. Don't know decimal stuff. Don't know how got 3000. … Then less because 49 x 58 equals more than 3000 so choice little less because 58 x 49 over 2000 but less than 3000.

Year 5

M Little less than 3000. Just guessed it. Did 49 x 58 in head. Did on top of each other [vertical form]. 200 something. Chose them because 2 one's before decimal.

D Chose little less than 3000. Wasn't too sure about it, just guessed because it would be too much 3000 and over. Sort of done this before – just a quick guess. [Stated did calculations in her head.]

G [Not happy about her choice and couldn't reason it out. Tried multiplying two numbers but had difficulty doing it because of the decimal points.] Tried to do 3 x 9 and 3 x 9 and 3 x 4 but then couldn't go any further. Originally chose c because just was feeling it was that.

Item 27

Year 7

Q [Initially said didn't know how worked out. Was quite frustrated by the question.] Maybe he did 9 x 3 and then 1 x 99.

V How 99 same number could have been multiplied by it. Steps: ones column, the tens. 9 x 13 is 117 plus 1170 as has place value of zero and then add.

Z [Initially said didn't know. Asked if could do it harder way knowing it would take longer than 3 seconds.] Found questions quite difficult. Have to write it down to get the correct answer.

Year 6

R If had a piece of paper, 9 x 3 = 27 , 9 x 10 = 90, 90 x 3 = 270, 90 x 10 = 900 and add all up = 1287.

M Couldn't come up with a way. If had a piece of paper, depends, if he had paper would have done it in columns.

A 99 same as 11 except 9. 11 x 13 = forgot something like 100 x something and then done something to make it 99. Plus it to make 1287 – not good at explaining. [A didn't actually work it out using her method but thought he might have used the way she explained.]

Year 5

M No clue how he could have done it. I would have done 99 x 13 written out in columns. Tried 99 x 13 in columns. It didn't work because it was slower than 3 seconds.

D Tried … could have done 99 x 13: 9 x 3 = 27 [and so on as used vertical form].

G He first did it [before he was challenged with the question – he had already worked out the answer]. Imagined equation going down in my head, 3 x 9 and then 1 x 9 + 2, 3 x 9 = 27 put down 7 and carry 2 … [explained process using vertical form].

Item 28

Year 7

Q No. [Why not?] Because when you take away 30 from 61 get 31. Then take away 1 and then get 30. That's where I am getting stuck.

V Got right because think 29 rounded to 30. Then 61 – 30 is 31, 1 too many because 29 rounded to 30 is 1 left. 61 – 30 = 31. Just have to add one more and he got it right.

Z Wrong, because 29 is less than 30. So answer to 61 – 29 is more than 30. So when take 1 away from 61 to 30 will be different answer. Think plus, but this is take away, plus leave over one. So 29 when minus it comes as a bigger number. 61 – 30 is smaller number as more than 29. If take one away because more smaller, but if take one away makes it smaller. Need to add one, not take away.

Year 6

R [Originally wrote 61 – 32 = 29 to prove correct.] Got muddled up. 30 and 31 – though took away 31 so had 30 and then said take away one more would leave 30 but he had to take away 2 more to get to 29.

M Right because 60 – 30 = 31 and then to take off one more would be 29 and that's the correct answer.

A No, because 61 – 29 does not equal 30 and he takes off one more. Worked 61 – 29 does not equal 30 but equals 31. He was meant to add one more instead of take away.

Year 5

M Reckon will get correct answer. Well, instead of taking one off 30, take off 60 so 60 – 30 = 30. Just instead of taking another one away take it from the 60.

D Because 61 – 30 is 31. There isn't one too many. He said there is one too many, it actually equals 32. Still not right. He is supposed to plus one not take away one.

G Wrong because not working out properly and he would get confused as I am. First he has like one too many. Doesn't know that he could work out the equation to work it out. Should go 61 – 29. Work out downwards on piece of paper, much easier.

Item 29

Year 7

Q Did first one 894 – 548 and got 346. Second one, 548 + 346 will show 894. I know, maybe they five you the number [346]. [Basically thought that for the second way of showing the answer maybe the question will be re-written to include the number 346 and not 548.]

V I did 894 – 548 because 894 seats and 548 were filled. Like this, [showed picture of graph and explained graph.] There is another way, yes it would take a long time. 548

+ x = 894. Go on forever but there is an answer you could estimate. Real thing, 548 + x = 894; 894-548 = x.

Z Well, there are 894 seats and only 548 filled. Obvious 548 filled so rest empty. Just minus 548 by 894. If check, 346 + 548 = 894.

Year 6

L Just get amount of seats and take away the other number 894. 548 filled and take away amount get the same answer.

X Could go in and actually count the seats. Wrote 894 and only 548 people in them. Took away from 894 so only 346 seats left. Added 548 + 346 to get how many seats in first place – but wouldn't know this.

F Just wrote 894 seats so 548 take away that. That's how many seats left. [Vertical form] just do it that way. Couldn't come up with anything great [second way prove answer]. Tried 548 + 894.

Year 5

A Don't think it is right. Added 548 to 12 to make 600 + 200 = 800 + 94 = 894. Answer 212. Second one [solution] not so well. 548 + 13 + 199 + 94 made up with 369. Only way know how to do it.

L Well, I had 894 and I took away 548 so I did 4 – 8 [explained vertical form process]. Chose way as easiest to do it. Once had answer put 346 + 548 = 894 and then had another problem 894 – 346. [Another solution given later on]: divide 548 by 894. That gave 100 and remainder 346. So that would be the answer.

S [Completed problem with vertical form but had difficulty with trading.] First cross out 8, didn't work decided cross out 9 and carry [vertical form]. Chose, probably one of ways to do it. Another way, 548 + 346 to check if I was right. [Another solution given later on]: 895 – 346 = 548.

Item 30

Year 7

B First add 36 and 48. Took away 6 and 8. 30 + 40 = 70. 10 + 8 = 78 and 78 + 6 = 84.

I Basically same way. 36 and 48. 48 and 36 added together. Just added together.

W Same. Added 30 and 40 first, = 70. 70 + 8 = 78 + 6 = 84. Pretty simple.

Year 6

L Wrote 36 and 48 down and added 8 and 6 which is 14 and carried. 4 and 4 is 8 = 84.

X Yes, because 36 and 48 is 84. Solved it in head. Added 4 and 3 is 70 and 8 and 6 is 14 and added together. To prove, write it down on sheet [vertical form].

F Just "plussed" it on paper. Quickest way to do it. Prove it by writing on paper 6 + 8 is 14. Bring 1 up then 4 + 4 is 8 so 84.

Year 5

A Yes. Just wrote it because did sum 36 + 48. Add 6 and 8, carry 1. Add 3 and 1 = 4 + 4 = 8 = 84.

L Did 6 + 8 = 14. Carried one to 3, trade it 4. 4 + 4 = 8 = 84. Came up another way: 4 – 6 couldn't do it.

S Because 6 + 8 = 14. carry one, 3 becomes 4 and 4 + 4 = 8 and = 84.

Item 31

Year 7

B Looked at it wrong. Said 812 – 379 = 433 [vertical form]. Not 481 so Mary wrong. Also + 1 when. Should have minus 1. Added extra 2 and made wrong.

I Thought that was smaller than that. The 380 + 1 bigger than 379. Don't think 481 is right. Just guessed wrong. Would have done 812 – 379 [vertical form].

W By looking at it. Answer 481 because 812 can start off with 800 – 300 = 500. Then 500 – 80 would make 420 + 1 = 421. First minus 812 – 300 = 512. Then minus 70. Then minus 9 = 433.

Year 6

L Way worked out, thought it was right. Hadn't worked it out first way. Worked out could have been right. Put plus 1 instead of – 1. To do it, 812 – 379 = 433 [vertical form]. Used fingers to count down.

X Had to work out, first didn't know she was right or wrong. Just looked funny - way wrote 2 equal signs. Just 812 – 379 = 433.

F Looked at hers. All confusing. Did own way. First 812 – 379. 812 – 380 + 1. Should have been minus 1 not + one.

Year 5

M Did same sum that Mary did. Took 9 away from 2 cross out 1 to make … [vertical form 812-379]. Take 379 away from 812 = 433.

L Subtracted 380 from 812 then added on 2. So double checked it: 810 – 380 = 330. Then added on 3 others which made 433. Checked it again, 812 – 379.

S Did 812 – 379. Knew it was wrong. [Used vertical form but was having difficulty with trading. S felt that Mary was wrong but couldn't prove it. In end stated that she was 50% sure Mary was wrong.]

Item 32

Year 7

B 8 + 8 = 16 so just minus 8, 1 so 7. So minus 1 on other 8 so 9. So pretty much same. Did with all the 7 + 7 = 14. 7 – 1 = 6 and 7 + 1 = 8. 16 + 16 = 32 minus 16 made 17. What left 15, so 15 + 17 = 32. 9 + 7, opposite, 9 – 1 add on to others = 8 + 8.

I Basically did the same. Added 1 or take 1 off and add to second or first number depending on equation.

W 8 + 8, basic so should know 9 + 7 = 6 so 6. 7 + 7 is 14. Try 9 + 8 = 17, 8 + 7 = 15, 8 + 6 = 14 by looking at knew answer. 16 + 16, saw 15 + 14 = 29, 15 + 16 = 31, 15 + 17 = 32. 15 + 15 is one to remember = 30. One down 15 + 14 = 29. One up 15 + 16 = 31. Two up = 32. 9 + 7: 9 + 9 = 18. 8 + 8 = 16. Pretty easy.

Year 6

L First one just know 8 + 8 = 16. Put 9 up to 10. Makes 17 and take away 1 is 16. Know 14 (7 + 7), 6 + 6 is 12, and 1 + 1 is 2 which is 32. I went 1 + 1 is 2 and 5 + 7 is [finger counting]

12 add on. 16, know that because 9. Just know 8 + 8 is 16.

X Just thought minus 1 from 8 is 7 and put on 8 is 9. Still using same number but changing around. Took one from 7 and put on 7 which is 8 and 6 which is 14. Did same took off [the 15] put on the other [16] which is 15 and 17. Other way round, took one off 9 and added 1 on 9.

F 8 + 8, just took 1 away from 9 and added 1 to 7. Same with second one. Add 1 to 6 and took away from 8. Just did it with 15 and 17. Did with all of them. Lowest one add on and highest one take one. 15 + 1, 17 – 1. Last one did it to, 9 + 7, 7 + 7, 9 – 1.

Year 5

M Knew 8 + 8 = 16. Added 3 out 9 on 7. That made 10 and added 6 left. So 16. 7 + 7 is 14 then take 1 off 8 makes other 7, makes 8 to 7 so 14. 16 + 16 = 32. Added 15 + 17. Took one off 17 to make 15 and 16. That made 17 and 16 = 32. 9 + 7 is 16. Just knew 8 + 8 = 16.

L Know 8 + 8 is same as 16. So looked through all answers. Added on 1 to all problems. So 10 + 6 = 6, took one away from 7. 7 + 7, know 7 x 2 is 14 and then 7 + 8 is 15. So made 6 + 8: made 6 the 7 and took 1 away and it made 14. 16 + 16, times 16 by 2 = 32 and I know 15 + 15 = 30. Made 14 into 15 and took 1 away = 29. Know 15 + 16 = 31 so c [multiple choice] must be the thing. 9 + 7 is 16 because up top [of page]. Obvious it is 8 + 8.

S Well, basically all do take 1 away from first number and add one to the first number. 7 + 7 = 8 + 6. Same on all other problems. 16 + 16 because 16 – 1 is 15 and 16 + 1 = 17. 9 + 7 is plus 1 on to 7 and take away 1 from 9 and come up with 8 + 8. Also know 16 because 8 + 8 is 16 and 9 + 7 is 10 + 7 – 1 = 16.

Item 33

Year 7

B Exactly same. 4 + 6 = 10, 5 + 5 = 10. 113 + 15 = 28, 14 + 14 = 28. So 13 + 13 = 26. 17 same as 9 + 8 = 17 because do 8 – 1 = 7. Put minus one on 9 makes 10 + 7. 28, same thing. 19 + 9, 19 = 20 + 8. 146, same, only with higher number.

I 4 + 6 same as 5 + 5 because take one off 6 add t 4. Basically did same for second one. Third one, just came to me. When I was small, always get wrong, now just remember 19 + 9. Took 1 off, made 20 + 8. 139 + 7, took 1 away from 7 added to first number.

W 6 + 4 is 10. So its same, basic, 5 + 5 is 10. 13 + 15, 28. So way can guess: 15 + 15 is 30 so minus 2 makes 28. 17 same 9 + 8. All time 9 plus 1 more than that. 28, 19 + 9, same plus 1. 146 add 146 + 1 = 146. Always add number 1 over if "plussing" with 9. 139, 140 + 6 = 146.

Year *6*

L Knew 4 + 6 = 10 and 5 + 5 = 10. Added ones = 20 and then 5 + 3 = 8. Looked at 14 + 14 because 4 + 4 = 8 and 1 + 1 = 20. Know 9 plus stuff. Take away, just know. You add 1 on 9 and just know it. 28, 19 + 8 – 1 less. Know 1 more would be right. 146 – 139 + 6 one less. So knew one more. 139 + 6, just knew because of the 9 thing. Just take away 1 from number makes 6. Add to 9 to make number.

X Just add 1 on 4, take 1 away from 6. Knew it but checked. 13 + 5: added 1 on 13 and 1 off 15. Worked out knew 13 and 15, 28. 14 + 14. Saw all 9's. Thought 9 + what = 17. I counted up to 17 but quickly in my head. Just knew 28 same as 19 + 9 because 19 close to 20, add 8 on 27. 9 on to 19 = 28. Forgot.

F Just added 1 to 4 and minus 1 off 6 so 5 + 5. 13 + 15: added one to 13. Took 1 off 15 so 14 + 14. 17 same as 9 + 8. Just add number on and takeaway from 7's so get 10 and take away ones. 28 plus 9 with it with 10 and take away 1 from one's column. Last one "plussed" 7 with 9 and said 10 instead 7 and took away from one again.

Year 5

A Know 4 and 6 is 10 so just added 5 and 5 = 10. Second one, 13 + 15, that's 28 and so knew 14 and 14 is 28. Add 10 and 10 makes 20 and add 4 and 4 = 8 = 28. Third one, 17, 9 and 8 = 17. Always knew that. 28 same as 19 and 7. Add one to 19 makes 20 and there was … opps wrong, it's 19 and 8 … no still wrong, don't know. 19 and 9, yeah 19 and 9. Add on 1 to 19 makes 20. 9 is 8, makes 28. Last one, 146, same as 134 + 7 because add one on makes 140 and 6 left over makes 146.

L 4 and 6 is 10. Know 5 x 2 is 5 + 5 = 10. So 13 + 15. I found that by adding 2 tens together which made it 20 and added 3 and 5. Made it 28. Went through all of them [problem choices] and added tens. 17 same as, added on 1 to all of them. Made it 16, 17, 18 and took 1 away. Started with, made 9 an 8 which made it 16 and added on 1 more, made it 17. 28: I added 9 and 9 made 18. Then 10 = 28. 146 – 139: made 140 + 6. Took one away = 145. Knew needed one more number so 7 + 139 must be it.

S 4 + 6 + 10 and worked out 5 and 5 is 10 because 5 + 5, ½ 10. 13 + 15 same as 14 + 14. 5 and 3 = 8 and 4 and 4 = 8 and 1 and 1 = 2. 17 same as 9 + 8, because 9 + 9 = 18 and take one away. 28 same as 19 + 7, no, not 19 + 7 it's 19 + 9 because 20 + 8 = 28 and 19 + 9 = 28. 146 same as 139 + 7 because exactly like 140 + 6.

Item 34

Year 7

B First, 67 + 34. Thought 105. Realised, so 101 so changed to correct. Checked 67 + 34 = 101. Another way 101 – 34 = 67. So 34 + 67 = 101.

I 7 + 4 carry 1 etc in my head but quickly. Another way, do it backwards doing subtracting or roughly out on paper.

W Did 100 – 30 makes 70. 70 – 4 = 66. Then minus the … whoa … 100 –34 = 66 then 1 left plus 1 makes 67. Another way 101 – 34 = 67.

Year 6

L Got 67 + 34. Wrote in sum .. 7 + 4 = 11 carry 1. 6 + 3 + 1 = 10. Just add 3 + 6, 4 + 7 and add 2 answers together. I just wrote answer and realized that was a way.

X 60 and 30 = 90. 7 and 3 is 10 + 1 = 11 and added together because 67 + 34 is 101.

F Because added numbers myself. 4 + 7 = 11. Carry 1 to 6 so 7 + 3 = 10. So 101. Another way for checking, use a calculator.

Year 5

A Added 7 and 4 = 11 then added 6 and 3 = 90. 60 and 30 = 90. Then added 11 to 90 = 10. Did it backwards for second one [part of question].

L Added 4 and 7 made 11. Carried 10 onto 6. Made 7. Added 3 to 7 = 101. Take 67 from 101 and = 34.

S 4 + 7 = 11 so put down the one and carried that other so that 6 became a 7 and 7 + 3 is 10 so the answer is 101. To check the answer: 101 – 34 which equals 67. Another way 101 – 67 = 34.

Item 35

Year 7

B Look a the question. A school has 5 classroom and 4 windows. Should be 20. Should be 4 x 5 = 20. [Asked to use children to write story problem not use windows as would be easier.] There were 5 children, 4 more came and how many were there now. 5 + 4 = 9.

I Because should be 5 x 4 = 20. Did 4 + 5 = 9 which is wrong.

W Wrong because there's 4 windows per 5. So group. Always times. 4 x 5 = 20 not 5 + 4 = 20.

Year 6

L Know wasn't correct because that would be for 1 classroom number. You have to times it for all the classrooms. Not right, just know have to do times so 5 x 4 = 20.

X No, because 5 and 4 is 9 but not 9 windows. Because added not times them. So 4 and 5 = 20.

F Wrong because plus. So if 5 classrooms and 4 windows just go 5 x 4 = 20.

Year 5

A Because 4 and 5 = 9 so yes, because says 5 and 4 = 9. [A asked at this point what did the word "windows" mean. Found out that A had just looked at the equation and failed to read the problem at the beginning.] Now I think it is wrong because if 5 classrooms and 4 windows in each, so 4 x 5 = 20.

L Read the question and looked at Jill's thing. Can't be right. 5 classrooms each and 4 windows. 5 + 4 can't be right. Has to be 5 x 4, that's 20 windows.

S If there were 5 classrooms and each had 4 windows. Her answer is wrong because did plus instead of times. 4 x 5 = 20 windows.

Item 36

Year 7

B Jim would have 1 more banana than Lee. Let's say Jim had 4 and Lee 1. 1 + 2 = 3. So Lee still less than Jim.

I Jim still more because if Jim had 6 Lee had 3. Even if gave him 2 more, Jim still have more than Lee. [Explanation]: Jim had 6 and Lee had 3 + 2 = 5.

W Jim 1 more than Lee. Would say if Jim had 4 and Lee 1 then 1 + 2 = 3 then, 1 + 3 = 4, wait … Jim still would have more. Jim 5, Lee 1 less than Jim. 1 + 3 = 4 then plus 1 = 5. Jim had 3 so plus 3 make 6 so 1, so Jim still more bananas more than 3.

Year 6

L Jim has one more banana because 3 – 2 = 1. That's how worked it out. Well Jim has 3 more bananas than Lee. Lee gets 2 and so 3 – 2 = 1. Jim had 3 and Lee 0, give Lee 2 then … didn't work out like that just know because 3 – 2 = 1.

X [At first mentioned didn't get the problem.] Because if Lee got 2 more bananas and Jim has 3 more and Lee has 2 Jim still has more because 3 bananas are more than 2 bananas. If Jim has 23 and Lee 20 and then 2 more have 22 but 23 is greater than 22.

F Jim has more because 3 odd number could never make equal unless chopped one banana in ½. If each already had banana gave Lee 2 and Jim 1 Lee would have most because Jim … wait, is these 3 bananas taking away and splitting. Jim still more because 3 smaller than 2. [Fixes diagram but still confusing.]

Year 5

A Say Tim and Lee each had bananas … They would only be 1 banana apart. Did it with pictures. Drew 3 bananas and 6 bananas. Lee had 3 + 2. Lee would have 5 and Jim would have 6.

L Jim had 3 more than3 if Lee got 2 more. Jim would have 1 more than Lee because Lee had 3 less than Jim and he got 2 more, not exactly 3. Put any number more. Jim could have 9 and Lee had 6. Wrote + 2 = 8. Jim still has 9. Jim has one more than Lee.

S Jim has one more than Lee. If he get's 2 more Jim more bananas. If Jim 12 and Lee 9, Lee gets 2 more makes 11. Jim has 3 more at beginning. Lee gets 2 but Jim has 1 more than Lee.

Item 37

Year 7

B Get 3 each, 1 person 3 etc. Last 2 apples cut into 1/5. So each child gets 3 and 2/5's apple each. Altogether 17 apples. Divided because can't give 2 apples to 5 people.

I Did 15 apples shared out. Tried working with 5, 3 and 2 first. Started working with number they show you. 17 apples altogether as 2 left over.

W Five children share some apples. Each get 3 because 3 x 5 = 15 + 2 makes 17 apples. Cut 2 apples in 1/5.

Year 6

L Just drew it. Lines for people. 3 each, 2 left over. Just showed it because didn't see bottom question. Counted 3, 6, 9, 15, and 2 = 17. Another way, 17 divided by 5. There's 3 remainder 2.

X If 5 children and get 3 each. 3 x 5 = 15 and 2 left over. 15 + 2 = 17. Picture to count all up. Put 2 in middle no where.

F Well, there were 5 kids each. One had 3 apples and 2 left over. No way sharing out. Add altogether 17. Pictures to show how thought. Draw 5 children. Each get 3. 2 left over.

Year 5

A Says 5 children 3 each [apples] and 2 leftover. Knew answer 17 so did 17 divided by 3 which is 5 remainder 2. How many left, 17. Figured out how can get 3 fives out of one number. Knew 15. So knew 2 left over. Knew to divide as quickest way.

L Know 3 x 5 is 15 and added on 2 more made 17. [Came to this conclusion after drawing out problem.] I drew 5 lots of 3 and made them apples and left out 2. Made lines through all 3's. Was 5 lots of 3, makes 15. Two left, 17.

S Drew 5 heads then gave 3 each with 2 apple on the side. How many: counted 3, 6, 9, 12, 15 + 2.

Item 38

Year 7

B Went in head, 3 + 5 + 7 = 14. Basic facts. Can break down 3 + 7 + 5 = 14 but that's wrong = 15. Wrote down, 3 + 5 + 7 = 15. Other way, 3 + 5 = 8, and 8 + 7 = 15.

I Did it like B. 3 + 5 + 7 = 15. Showed 14 which is wrong. 3 + 5 = 8, 8 + 7 = 15.

W 3 + 5 + 7 = 14 but wrong because 5 + 3 = 8 and 8 + 7 = 15. Checked it in my mind. 5 + 3 = 8 then + 7 = 15 not 14. Other way, 15 – 7 = 8 and 8 + 7 = 15.

Year 6

L Just did 5 + 3, oops did it wrong. 5 + 3 is 8 and 7 + 7 = 14. Add 1 is 15. Thought right but then 8 + 7 does not equal 14 equals 15. Another way, do it back to front. 7 + 5 + 3 = 15.

X 5 + 3 = 8 not 7. 7 + 7 = 14. Need 7 + 7 not 7 + 7 because 7 and 8 = 15.

F 5 + 3 = 8 and 8 + 7 = 15. But so, if did 7 + 7 = 14. Basically another way 7 + 7 = 14.

Year 5

A Yes, 3 and 5 makes 8 and 8 and 8 = 16. So take away 1 makes 8 and 7. No … wait .. is wrong. Just 3 and 5 is 8 + 7 that's more than 14, so 15. Other way 5 and 7 + 3 = 15. 8 + 7, so 8 + 8 = 16 – 1 + 15.

L Well, no. I did 3 + 5 = 8 and 8 + 7 = 15. Another way, 14 – 8 = 6. Wrong, so I put a cross through it 15 = 8 = 7. Put a tick [next to her correct equation]. 8 + 7, know 7 + 7 is 14 + 1 = 15.

S Said no. 5 + 3 = 8 and 8 + 7 = 15. So the answer was wrong. To check, 14 – 8 = 6, couldn't equal 14. 8 + 7 is 8 + 8 = 16 take away 1 = 16.

Item 39

Year 7

B Not right. Might have multiplied together but heaps more than 410. Might have gone 2 + 2 = 4 and 7 + 3 = 10 and put together, 410. Probably how did it because student is Year 3 and can't do times.

I Thought copied down wrong. No way, by adding or multiplying to get 23 x 27 to go into 410. Maybe copied it down wrong.

W Well dumb if do it my way which was 180 + 230 = 410. Pretty dumb is she did it my way.

Year 6

L Think she did 7 + 3 = 10 and worked out and wrote down and did 2 + 2 is 4 and put 4 in front of 10.

X [Needed prompting to come up with a way. Couldn't think of one. Suggested may have multiplied it.] Just seeing if she multiplied it.

F Reckon if do answer 7 + 3 is 10. Must have gone 10 + another 30 and added and accidental zero so 410.

Year 5

A Think wrong. Two numbers that small can't equal that number. She made 2 two's 200. Made 7 and 3 tens. So made 200 + 200 = 400 and 7 + 3 = 10 so 410.

L Well, knew wrong. 20 + 20 = 40. So I wrote down then 7 + 3 = 10. Put 10 and 40 together so = 50. She made 20's 100's, made them 200. So made 400 and 7 + 3 would be 10 so 410.

S Thought wrong. Could have thought 270 + 230 which is wrong anyway because = 500. Could have thought 207 + 203 which is 410 instead of doing 27 + 23 which = 50.